

A Framework for Evaluating Design and Implementation of Activities for Mathematics Instruction

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FOREWORD

In the realm of mathematics education, considerable attention is given to the methods and quality of instruction. One focal point that has garnered significant research interest revolves around the introduction of an instructional approach that facilitates meaningful structuring of learning experiences and ensures their lasting effectiveness. An instrumental methodology that has risen to prominence within this context is activity-based teaching.

When we delve into the literature pertaining to activity-based teaching in mathematics education, we observe that numerous studies have been conducted on the design, implementation, and evaluation of activities. However, there is a notable gap that structured methodologies which can be employed to evaluate the design and implementation of activities are lacking. Hence, there is a critical need to develop a practical tool built on theoretical foundations. This tool would serve the dual purpose of assessing the quality of activity-based learning efforts and offering feedback to practitioners, while also guiding the intricate processes of activity design and implementation in a comprehensive manner.

In this book, we present a tool with the aim of providing practical insights to both practitioners and researchers. This tool, referred to as the “Framework for Mathematical Activities” (FfMA), serves as a guide for evaluating the quality of activity design and implementation. We have employed a design-based research approach and have demonstrated its functionality based on evidence. Through this approach, we primarily modeled activity-based teaching grounded in design and practice, which then guided the creation of FfMA.

During the development of FfMA, we recognized the importance of establishing performance indicators for its components. These indicators facilitate the evaluation of activity script and implementation processes, which are essential for activity-based instruction. As a result, we assigned grades to these components and formulated criteria. FfMA, whose effectiveness has been evidenced, provides users with the opportunity to evaluate the process of activity-based instruction through a structured and analytical approach.

We would like to thank our colleagues who actively participated in the workshops and focus group meetings conducted under the project, representing a diverse array of universities. Their insights and contributions have been invaluable in shaping the trajectory of this collaborative endeavor. Indeed, this book stands as a testament to the strength of teamwork.

We further wish to express our thanks to our doctoral students Gülbahar Bakırcı and Sibel Tutan, as well as graduate student Taha Memiş, whose

involvement and contributions within the project's framework have been pivotal. Their dedicated efforts have greatly enriched the studies conducted.

Lastly, our deep gratitude goes to the teachers who partnered with us during the actual classroom piloting of FfMA. Their on-the-ground experiences and cooperation have provided vital perspectives that have enhanced the practicality and efficacy of this tool.

Authors

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We also would like to express that Prof. Dr. Mehmet Fatih Özmantar undertook the substantial task of preparing the English version of this book. He ensured that the core essence and the intricate nuances of the text, originally crafted in Turkish, were faithfully retained in the translation process. His commitment to preserving the integrity of this work has been instrumental in making this book accessible to a broader readership.

TABLE OF CONTENTS

Authors	iii
Foreword.....	v
Acknowledgement.....	vii
Chapter 1: Introduction	1
Chapter 2: The Role of Activities in Mathematics Education.....	7
2.1. The Concept of Activity and Its Use in Mathematics Education	7
2.2. Rich Mathematical Tasks.....	10
2.3. Components of Design and Implementation of Activities.....	14
2.4. Evaluation Approaches to Activity Design and Implementation Processes.....	21
Chapter 3: Development of The Mathematical Activity Evaluation and Feedback Tool	23
3.1. Stage One: Defining the Problem.....	24
3.2. Stage Two: Identifying the Dimensions and Indicators of FfMA	25
3.3. Stage Three: Creating the First Version of FfMA	26
3.4. Stage Four: Successive Efforts for Improvement	27
3.5. Stage Five: Finalising the FfMA.....	29
3.6. Validity, Reliability and Usability of the FfMA.....	29
Chapter 4: Mathematical Activity Design and Implementation Model: A Conceptual Framework.....	33
4.1. Operational Definition of Mathematical Activity	33
4.2. Mathematical Activity Design and Implementation Model.....	35
Chapter 5: Mathematical Activity Evaluation and Feedback Tool.....	39
5.1. FfMA and Its Structural Characteristics	39
5.2. Dimensions and Components of FfMA.....	41
5.2.1. Components of Activity Script.....	41
5.2.2. Components of Activity Implementation.....	52
5.2.3. Components of Mathematical Potential	73

Chapter 6: Considerations in Using FfMA	83
6.1. Considerations in Using FfMA as an Evaluation Tool.....	84
6.2. Considerations in Using FfMA as a Feedback Tool.....	86
Chapter 7: Contribution of FfMA to The Field of Mathematics Education	83
7.1. Importance of FfMA.....	87
7.2. Potential Users of FfMA.....	90
7.3. Recommendations for Further Research.....	91
References	93
Appendices	99
Appendix 1: Indicators for Evaluation of Activity Script.....	99
Appendix 2: Indicators for Evaluation of Activity Implementation	100
Appendix 3: Indicators for Evaluation of Mathematical Potential of Activity Script and Implementation.....	102
Appendix 4: Criteria for Evaluation of the Activity Script Components.....	103
Appendix 5: Criteria for Evaluation of Activity Implementation Components.....	105

CHAPTER 1

INTRODUCTION

Recent research in mathematics education has increasingly focused on the quality of teaching methods. The aim has been to foster meaningful learning experiences and develop teaching strategies that promote deep understanding of mathematical ideas. Within this context, activity-based teaching has emerged as a preferred approach among mathematics educators. The International Commission on Mathematical Instruction (ICMI), a leading organization in the field, highlighted the importance of this approach in 2015. Following their exploration of related studies, ICMI published a volume on “Task Design in Mathematics Education” to guide further research (Watson & Othani, 2015). The diverse backgrounds and roles of the book’s contributors, ranging from designers to practitioners and developers, clearly indicates the global interest and emphasis on activity-based teaching in mathematics education. This approach is embraced primarily because it encourages students to take ownership of their own learning, fostering a deeper and richer comprehension of mathematical concepts (Lozano, 2017).

While there may be various definitions for what constitutes a ‘mathematical activity’ in the literature (Bozkurt, 2012; Margolinas, 2013; Özgen, 2017; Özmantar et al, 2010; Stein & Smith, 1998), a common thread among these definitions is the structuring of activities around a mathematical task. Doyle (1988) offers a framework for understanding what constitutes an academic task, delineating four key components that are applicable across different fields of instruction. These components are:

- *Responsibility* – pertains to the level of accountability expected from the learner,
- *Operations* – describe the procedures to be followed,
- *Resources* – involve the tools or materials required,
- *The product* – refers to the expected outcome of the task.

The effective use of a task for educational purposes is not solely determined by the task itself but is influenced by a multitude of factors. These can include the instructional perspective of the teacher, the underlying values that guide the teaching process, and the particular pedagogical approaches the teacher employs to make the content accessible and understandable to students (Watson, 2008). Such factors are intimately tied to the teacher's own understanding of pedagogy. Hence, when a mathematical task is integrated into classroom instruction in alignment with a specific pedagogical approach, some researchers (Jones & Pepin, 2016; Özmantar et al., 2010) term this confluence of task and pedagogy as a 'mathematical activity.

Utilizing mathematical activities in educational settings empowers students to engage in various intellectual undertakings. These include exploring and understanding new situations, forming independent inferences and hypotheses, conducting alternative solution experiments, and critically discussing their findings (Choy, 2016; Driver & Oldham, 1986). Beyond simply serving a procedural function, instructional activities convey deep insights into the nature of learning, positioning students as active architects of their own developmental trajectories.

Corroborating this observation, Watson and Othani (2015) assert that activities are instrumental in facilitating students' engagement with mathematical concepts, ideas, and strategic thinking. The authors emphasize that instructional activities enhance students' ability to appropriate a mathematical worldview, apply it contextually, and cultivate advanced mathematical understandings. This view is supported by Lozano (2017), who posits that activities significantly influence students' mathematical thinking. Additionally, these activities provide critical insights into the essence of mathematics, its practicality, and the overarching goals of mathematics education (Henningsen & Stein, 1997, p.525).

Empirical research highlights the effectiveness of activity-based teaching. Numerous studies have consistently shown that active student participation in the learning process not only improves their understanding but also notably enhances their performance in mathematics (Agyei & Voogt, 2016; Erdem & Aktaş, 2018; Gürbüz et al., 2010; Pokhrel, 2018). Nonetheless, to fully harness the advantages documented in these studies, it is crucial that the design and implementation of mathematical activities in instructional settings ought to be thoughtfully and carefully crafted.

Activity design serves as the initial stage of preparation where activities are selected and planned for implementation in an educational setting. Quality design entails the identification of potential challenges and the provision of pre-emptive

solutions that might arise during implementation (Griffin, 2009). The context of this design is often shaped by the nature of the chosen mathematical tasks. Boaler (1993), for instance, posits that activities should incorporate real-world problems, while Baki et al. (2009) stress the significance of non-routine problems. Özmantar and Bingölbali (2009) advocate for enhancing activities with supplementary materials and emphasize deploying mathematically-rich tasks. Moreover, effective implementation of these activities is essential for realizing their full educational benefits. Critical variables such as classroom management, time allocation, and the proper execution of instructions have been highlighted in the literature as key determinants of success (Swan, 2007; Horoks & Robert, 2008). Although there are many design and implementation characteristics essential for the success of activity-based mathematics instruction, current research has not converged into a clear set of guidelines for practitioners. Frequently, the criteria pinpointed by scholars lean more towards academic exploration than practical use. As a result, while these quality-assurance criteria are empirically validated, they often remain within the realm of academic research. There exists a gap in the literature for a comprehensive framework that can guide practitioners and ensure quality implementation in a holistic manner.

Quality assurance in activity-based mathematics instruction requires a thorough examination of both its design and execution. Central to this endeavour is the evaluation process. This process is crucial for gauging the merit of the activity's design and its execution, pinpointing areas for improvement, and steering decision-making (Liljedahl et al, 2007). However, evaluation alone is not enough to raise the bar on quality; it needs to be complemented by insightful feedback. Feedback, as Wiggins (2012) elucidates, offers insights about specific behaviours and actions. Broadly, it is information regarding one's performance or understanding (Hattie & Timperley, 2007). It provides clarity on how one has performed, offering a route to fine-tune one's approach and address any gaps. Feedback is paramount not just for personal growth but also as a directive for enhancing performance (Molloy & Bound, 2013). Incorporating feedback into the evaluation process equips teachers with the tools to enhance the caliber of activity-based teaching strategies. Furthermore, it empowers them with self-regulatory skills, stemming from increased awareness.

An examination of the existing literature reveals multifaceted discussions surrounding the design and implementation of activities. Topics range from understanding activity as a concrete entity (Doyle, 1988; Uğurel & Bukova-Güzel, 2010), the essence of activity-based mathematics education (Olkun & Toluk, 2005), the pivotal role of activities in facilitating conceptual learning (Jaworski,

2004; Simon & Tzur, 2004), to principles governing the design and execution of activities (Ainley, Pratt & Hansen, 2006; Özmantar & Bingölbali, 2009). Yet, a closer examination of these studies highlights a conspicuous absence: there is no comprehensive framework available for evaluating activity-based teaching. Furthermore, there is a lack of guidance for teachers on delivering feedback grounded in quantifiable indicators.

In recent decades, a major thrust in educational research, particularly within mathematics education, has been the formulation of structured evaluation frameworks to assess various components of the instructional process. Since the early 2000s, several noteworthy frameworks have emerged. For instance, CRESST (The National Centre for Research on Evaluation, Standards, Student Testing) introduced by Clare (2000) offers another influential model. Additionally, frameworks like MQI (Mathematical Quality of Instruction) presented by Hill and colleagues (2008) have gained recognition. TRU (The Teaching for Robust Understanding) developed by Schoenfeld (2013) assesses the quality of instruction by highlighting both the strengths and limitations of classroom practices. While these theoretical models differ in their specific perspectives and approaches, they share a unifying characteristic: they all are structured to present dimensions of classroom practice. These dimensions are organized around specific indicators, providing practical insights that assist practitioners. Furthermore, they offer feedback mechanisms through systematic evaluations.

One might wonder why, given the existence of such comprehensive frameworks, there remains a void in evaluation models specifically tailored for assessing activity design and the implementation process. When examining the current body of research concerning the design and execution of activity-based instruction, we have identified several factors which appear to contribute to this gap as follows:

Theoretical Over Emphasis: Research concerning activity design and its implementation predominantly operates at a theoretical level, with insufficient transition to practical applications.

Lack of a Holistic Approach: Many researchers have tackled activity design and its implementation in a fragmented manner, rather than adopting a comprehensive and integrated perspective.

Absence of a Guiding Framework: To date, there has not been a development of a theoretical framework specifically tailored to evaluate and provide feedback on the quality of activity design and its subsequent implementation.