A Framework for Evaluating Design and Implementation of Activities for Mathematics Instruction

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FOREWORD

In the realm of mathematics education, considerable attention is given to the methods and quality of instruction. One focal point that has garnered significant research interest revolves around the introduction of an instructional approach that facilitates meaningful structuring of learning experiences and ensures their lasting effectiveness. An instrumental methodology that has risen to prominence within this context is activity-based teaching.

When we delve into the literature pertaining to activity-based teaching in mathematics education, we observe that numerous studies have been conducted on the design, implementation, and evaluation of activities. However, there is a notable gap that structured methodologies which can be employed to evaluate the design and implementation of activities are lacking. Hence, there is a critical need to develop a practical tool built on theoretical foundations. This tool would serve the dual purpose of assessing the quality of activity-based learning efforts and offering feedback to practitioners, while also guiding the intricate processes of activity design and implementation in a comprehensive manner.

In this book, we present a tool with the aim of providing practical insights to both practitioners and researchers. This tool, referred to as the "Framework for Mathematical Activities" (FfMA), serves as a guide for evaluating the quality of activity design and implementation. We have employed a design-based research approach and have demonstrated its functionality based on evidence. Through this approach, we primarily modeled activity-based teaching grounded in design and practice, which then guided the creation of FfMA.

During the development of FfMA, we recognized the importance of establishing performance indicators for its components. These indicators facilitate the evaluation of activity script and implementation processes, which are essential for activity-based instruction. As a result, we assigned grades to these components and formulated criteria. FfMA, whose effectiveness has been evidenced, provides users with the opportunity to evaluate the process of activity-based instruction through a structured and analytical approach.

We would like to thank our colleagues who actively participated in the workshops and focus group meetings conducted under the project, representing a diverse array of universities. Their insights and contributions have been invaluable in shaping the trajectory of this collaborative endeavor. Indeed, this book stands as a testament to the strength of teamwork.

We further wish to express our thanks to our doctoral students Gülbahar Bakırcı and Sibel Tutan, as well as graduate student Taha Memiş, whose involvement and contributions within the project's framework have been pivotal. Their dedicated efforts have greatly enriched the studies conducted.

Lastly, our deep gratitude goes to the teachers who partnered with us during the actual classroom piloting of FfMA. Their on-the-ground experiences and cooperation have provided vital perspectives that have enhanced the practicality and efficacy of this tool.

Authors

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We also would like to express that Prof. Dr. Mehmet Fatih Özmantar undertook the substantial task of preparing the English version of this book. He ensured that the core essence and the intricate nuances of the text, originally crafted in Turkish, were faithfully retained in the translation process. His commitment to preserving the integrity of this work has been instrumental in making this book accessible to a broader readership.

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CHAPTER 1

INTRODUCTION

Recent research in mathematics education has increasingly focused on the quality of teaching methods. The aim has been to foster meaningful learning experiences and develop teaching strategies that promote deep understanding of mathematical ideas. Within this context, activity-based teaching has emerged as a preferred approach among mathematics educators. The International Commission on Mathematical Instruction (ICMI), a leading organization in the field, highlighted the importance of this approach in 2015. Following their exploration of related studies, ICMI published a volume on "Task Design in Mathematics Education" to guide further research (Watson & Othani, 2015). The diverse backgrounds and roles of the book's contributors, ranging from designers to practitioners and developers, clearly indicates the global interest and emphasis on activity-based teaching in mathematics education. This approach is embraced primarily because it encourages students to take ownership of their own learning, fostering a deeper and richer comprehension of mathematical concepts (Lozano, 2017).

While there may be various definitions for what constitutes a 'mathematical activity' in the literature (Bozkurt, 2012; Margolinas, 2013; Özgen, 2017; Özmantar et al, 2010; Stein & Smith, 1998), a common thread among these definitions is the structuring of activities around a mathematical task. Doyle (1988) offers a framework for understanding what constitutes an academic task, delineating four key components that are applicable across different fields of instruction. These components are:

- *Responsibility* – pertains to the level of accountability expected from the learner,
- *Operations* – describe the procedures to be followed,
- *Resources* – involve the tools or materials required,
- *The product* – refers to the expected outcome of the task.

The effective use of a task for educational purposes is not solely determined by the task itself but is influenced by a multitude of factors. These can include the instructional perspective of the teacher, the underlying values that guide the teaching process, and the particular pedagogical approaches the teacher employs to make the content accessible and understandable to students (Watson, 2008). Such factors are intimately tied to the teacher's own understanding of pedagogy. Hence, when a mathematical task is integrated into classroom instruction in alignment with a specific pedagogical approach, some researchers (Jones & Pepin, 2016; Özmantar et al., 2010) term this confluence of task and pedagogy as a 'mathematical activity.

Utilizing mathematical activities in educational settings empowers students to engage in various intellectual undertakings. These include exploring and understanding new situations, forming independent inferences and hypotheses, conducting alternative solution experiments, and critically discussing their findings (Choy, 2016; Driver & Oldham, 1986). Beyond simply serving a procedural function, instructional activities convey deep insights into the nature of learning, positioning students as active architects of their own developmental trajectories.

Corroborating this observation, Watson and Othani (2015) assert that activities are instrumental in facilitating students' engagement with mathematical concepts, ideas, and strategic thinking. The authors emphasize that instructional activities enhance students' ability to appropriate a mathematical worldview, apply it contextually, and cultivate advanced mathematical understandings. This view is supported by Lozano (2017), who posits that activities significantly influence students' mathematical thinking. Additionally, these activities provide critical insights into the essence of mathematics, its practicality, and the overarching goals of mathematics education (Henningsen & Stein, 1997, p.525).

Empirical research highlights the effectiveness of activity-based teaching. Numerous studies have consistently shown that active student participation in the learning process not only improves their understanding but also notably enhances their performance in mathematics (Agyei & Voogt, 2016; Erdem & Aktaş, 2018; Gürbüz et al., 2010; Pokhrel, 2018). Nonetheless, to fully harness the advantages documented in these studies, it is crucial that the design and implementation of mathematical activities in instructional settings ought to be thoughtfully and carefully crafted.

Activity design serves as the initial stage of preparation where activities are selected and planned for implementation in an educational setting. Quality design entails the identification of potential challenges and the provision of pre-emptive solutions that might arise during implementation (Griffin, 2009). The context of this design is often shaped by the nature of the chosen mathematical tasks. Boaler (1993), for instance, posits that activities should incorporate real-world problems, while Baki et al. (2009) stress the significance of non-routine problems. Özmantar and Bingölbali (2009) advocate for enhancing activities with supplementary materials and emphasize deploying mathematically-rich tasks. Moreover, effective implementation of these activities is essential for realizing their full educational benefits. Critical variables such as classroom management, time allocation, and the proper execution of instructions have been highlighted in the literature as key determinants of success (Swan, 2007; Horoks & Robert, 2008). Although there are many design and implementation characteristics essential for the success of activity-based mathematics instruction, current research has not converged into a clear set of guidelines for practitioners. Frequently, the criteria pinpointed by scholars lean more towards academic exploration than practical use. As a result, while these quality-assurance criteria are empirically validated, they often remain within the realm of academic research. There exists a gap in the literature for a comprehensive framework that can guide practitioners and ensure quality implementation in a holistic manner.

Quality assurance in activity-based mathematics instruction requires a thorough examination of both its design and execution. Central to this endeavour is the evaluation process. This process is crucial for gauging the merit of the activity's design and its execution, pinpointing areas for improvement, and steering decision-making (Liljedahl et al, 2007). However, evaluation alone is not enough to raise the bar on quality; it needs to be complemented by insightful feedback. Feedback, as Wiggins (2012) elucidates, offers insights about specific behaviours and actions. Broadly, it is information regarding one's performance or understanding (Hattie & Timperley, 2007). It provides clarity on how one has performed, offering a route to fine-tune one's approach and address any gaps. Feedback is paramount not just for personal growth but also as a directive for enhancing performance (Molloy & Bound, 2013). Incorporating feedback into the evaluation process equips teachers with the tools to enhance the caliber of activity-based teaching strategies. Furthermore, it empowers them with self-regulatory skills, stemming from increased awareness.

An examination of the existing literature reveals multifaceted discussions surrounding the design and implementation of activities. Topics range from understanding activity as a concrete entity (Doyle, 1988; Uğurel & Bukova-Güzel, 2010), the essence of activity-based mathematics education (Olkun & Toluk, 2005), the pivotal role of activities in facilitating conceptual learning (Jaworski, 2004; Simon & Tzur, 2004), to principles governing the design and execution of activities (Ainley, Pratt & Hansen, 2006; Özmantar & Bingölbali, 2009). Yet, a closer examination of these studies highlights a conspicuous absence: there is no comprehensive framework available for evaluating activity-based teaching. Furthermore, there is a lack of guidance for teachers on delivering feedback grounded in quantifiable indicators.

In recent decades, a major thrust in educational research, particularly within mathematics education, has been the formulation of structured evaluation frameworks to assess various components of the instructional process. Since the early 2000s, several noteworthy frameworks have emerged. For instance, CRESST (The National Centre for Research on Evaluation, Standards, Student Testing) introduced by Clare (2000) offers another influential model. Additionally, frameworks like MQI (Mathematical Quality of Instruction) presented by Hill and colleagues (2008) have gained recognition. TRU (The Teaching for Robust Understanding) developed by Schoenfeld (2013) assesses the quality of instruction by highlighting both the strengths and limitations of classroom practices. While these theoretical models differ in their specific perspectives and approaches, they share a unifying characteristic: they all are structured to present dimensions of classroom practice. These dimensions are organized around specific indicators, providing practical insights that assist practitioners. Furthermore, they offer feedback mechanisms through systematic evaluations.

One might wonder why, given the existence of such comprehensive frameworks, there remains a void in evaluation models specifically tailored for assessing activity design and the implementation process. When examining the current body of research concerning the design and execution of activity-based instruction, we have identified several factors which appear to contribute to this gap as follows:

Theoretical Over Emphasis: Research concerning activity design and its implementation predominantly operates at a theoretical level, with insufficient transition to practical applications.

Lack of a Holistic Approach: Many researchers have tackled activity design and its implementation in a fragmented manner, rather than adopting a comprehensive and integrated perspective.

Absence of a Guiding Framework: To date, there has not been a development of a theoretical framework specifically tailored to evaluate and provide feedback on the quality of activity design and its subsequent implementation.

Lack of Performance Indicators: There is a notable absence of established performance indicators intended to assess and give feedback on the quality of both the design and the execution of activities.

Insufficient Practical Guidance: There is a shortage of research that serves dual purposes: guiding teachers in the trenches of day-to-day instruction and offering tangible evaluations and feedback on activity-based teaching. This guidance should ideally be rooted in actual classroom practices, providing evidence-based strategies and insights.

In essence, while there is recognition of the pivotal role of activities in enhancing mathematical understanding, the field lacks concrete tools and guidance for teachers to design, implement, evaluate, and refine these activities in actual settings. To address the gaps identified in existing literature, there emerged a clear necessity for the development of a practical tool established on solid theoretical foundations. Such a tool should provide insights into the quality of activity-based teaching, offer constructive feedback to teachers, and comprehensively structure the design and implementation processes for educational activities.

In pursuit of this goal, a project was initiated, funded by TUBITAK (Project Number: #119K773). Spanning two years from 2020 to 2022, this project successfully led to the development of the "Framework for Mathematical Activities" (FfMA). The FfMA serves as a robust framework for evaluating the quality of both the design and implementation phases of instruction activities in mathematics. This book aims to delve into the theoretical, methodological, and structural foundations of FfMA, providing readers with a comprehensive understanding of its origins, objectives, and applications.

Following this introductory chapter, the second chapter of the book offers an extensive literature review centered on the notion of instructional activities in mathematics education. This chapter examines the role of activities, explores various definitions, and discusses factors that contribute to their quality. Through this review, we aim to accomplish two key objectives: First, to illuminate the necessity for the development and application of the FfMA; and second, to present a concise overview of existing research findings pertinent to this endeavour.

The third chapter delves into the research methodology employed to develop the FfMA. Specifically, the chapter outlines the application of design-based research, an iterative approach that involves a collaborative effort between researchers and practitioners to develop practical, yet theoretically grounded products. This chapter elucidates the process through which design-based research was utilized in the creation of the FfMA, offering insights into how this methodological approach contributed to the framework's practical utility and theoretical foundation.

The fourth chapter attends to two essential theoretical perspectives foundational to the development of FfMA. The first section provides an operational definition of the term "activity," detailing its specific connotation within the context of this work. This delineation is necessary for comprehending the broader nuances of the FfMA. The subsequent section expounds on a comprehensive model, which systematically governs the design and implementation of mathematical activities. These core ideas are central to the FfMA's foundation and directly influence how we assess and provide feedback.

In the fifth chapter, the FfMA is detailed using a structured rubric format. Within this chapter, we provide a concise overview of the fundamental structural characteristics of the FfMA. This is followed by an exposition on the dimensions, components, content, and specific indicators associated with each component. To supplement this, a comprehensive rubric is presented, wherein performance levels are rated based on predetermined standards for every respective component.

Chapter six focuses on the utilization of the FfMA as both an evaluation and feedback tool. This chapter highlights the key considerations one must take into account when applying the FfMA for assessment purposes. Additionally, the chapter delves into the significance of employing FfMA-generated feedback for enhancing teaching practices, underlining the value of the insights gained from this tool.

The seventh and last chapter of the book emphasise both the practical and theoretical contributions that the FfMA aims to bring to the discipline. The relevance and importance of FfMA are examined in relation to existing literature. To elucidate its practical utility, we outline potential users of the tool and provide guidance on its application in real-world settings. The chapter concludes by offering suggestions for future research endeavours centred around the FfMA.

CHAPTER 2

THE ROLE OF ACTIVITIES IN MATHEMATICS EDUCATION

In this chapter, we aim to offer a comprehensive review of research studies carried out about activities in mathematics education that underpin the development of FfMA. We begin by highlighting the significance of activities in mathematics education and delve into how different researchers approach this concept. Subsequently, we explore the notion of “rich mathematical activity”—a term coined to underscore the mathematical essence of learning objectives targeted by the activities. We then consider the quality of activity design and explore elements that influence the effectiveness of its implementation. Finally, the approaches developed for the evaluation of activity design and implementation are shared with the reader.

2.1. The Concept of Activity and Its Use in Mathematics Education

The idea of integrating activities into mathematics instruction has captured the attention of both practitioners and researchers for many years. This growing interest can be attributed to the shift from traditional teaching methods towards learner-centred approaches. Roger and Freiberg (1994) argue that in learner-centred environments, both teachers and students share the responsibility of learning. While teachers are tasked with organizing the learning environment and supplying necessary resources, students are encouraged to take the initiative to direct their own learning. Roger and Freiberg further contrast these two approaches, noting that while learner-centred methods cultivate students to be independent and democratic thinkers, traditional ones merely condition them to comply with authority. From this viewpoint, it becomes evident that activities hold significant value in educational contexts. This is because activity-based mathematics instruction emphasizes student-centred engagement, where students actively participate in and take ownership of their own learning.

While activity-based teaching is not a standalone theory or model, it encompasses teaching practices where activities play an essential role in both the planning and execution phases. Given its emphasis on student ownership of learning, this approach is commonly linked to constructivist theories. For example, structured applications in realistic mathematics education often incorporate activity-based teaching, as evidenced by studies like those of Van den Heuvel-Panhuizen and Drijvers (2020). Likewise, many theoretical frameworks, including the French Didactical Engineering Theory (Artigue, 2009) and Lesson study (Doig et al, 2011), integrate activities as core components. Thus, activities hold a significant role not just in theory, but also in actual teaching practices. In this respect, it is fairly evident that the activities have a place beyond the theory in teaching and find a wide area of use in practice.

In mathematics education, the term "activity" frequently emerges in both research and everyday teaching practice. Many researchers in mathematics education have examined the concept of activity in connection to mathematical tasks. Such a task could be a problem to solve, a project to complete, a worksheet of exercises, or even more extensive undertakings. Building on this notion, Doyle and Carter (1984), followed by Doyle (1988), identified four fundamental components of an academic task: responsibility, operations, resources, and product. They essentially defined an academic task as an active process, using specific resources, to generate a significant product. Based on the concept of a task, Watson (2008) underlined that an activity is in-class practice which lacks meaning on its own but becomes meaningful when it is performed with the students under teacher guidance. This perspective accentuates the interplay between the activity and the teaching method applied. Echoing this perspective, Özmantar and Bingölbali (2009) described an activity as the practical execution of learning tasks, shaped by a distinct pedagogical strategy.

In the relevant literature, the term "activity" has been defined and conceptualized in various ways. Stein et al. (1996, p.460) consider an activity as classwork that directs students' attention towards particular mathematical ideas. Becker and Shimada (1997, p.6) describe it as environments crafted for undertaking intricate mathematical tasks that necessitate the employment of specific materials. According to Herbst (2008) activity is a structure employed during the actions and interactions of a group, demanding the use of certain resources. Özmantar et al. (2010) perceive it as engaging and captivating educational tasks that demand active student involvement. These activities assign responsibilities, encompass actions executed with certain tools and resources, and aim at generating a specific product to meet predetermined outcomes. Toprak et al. (2014) define it as a learning segment encompassing both planning and execution phases and necessitating social interactions within the classroom setting. Öztürk and Işık (2020), on the other hand, describe it as a pre-planned learning process that takes place under the control of a guide. In this process students actively engage, fostering social interactions, with anticipated positive outcomes at its conclusion.

From the definitions presented, it is evident that the term "activity" in educational contexts, especially in mathematics, is multifaceted and varies in its nuances. Various researchers have identified diverse characteristics that an effective activity should embody. Some key qualities emphasized in the literature are described as follows:

1. Relevance and Engagement: Activities should captivate students' attention and interest. They need to be relatable to real-life situations and be student-centric. This ensures that learners are motivated and can relate their learning to practical contexts (Bukova Güzel & Alkan, 2005).

2. Constructive Learning: Activities should be structured such that students build upon their prior knowledge, leading to the construction of new understanding. This approach should further empower them to apply the newly acquired knowledge in different contexts, promoting a deeper understanding (Gömleksiz, 2005; Hugener et al., 2009).

3. Flexibility in Implementation: Effective activities can be adaptable to both individual and group settings. This versatility allows teachers to tailor their approach based on the learning objective and the dynamic of the class (Baki, 2008).

4. Cognitive Engagement: Activities should challenge students cognitively. They must involve tasks that require the use of mathematical notations and symbols, modelling, logical reasoning, and abstraction. Such processes stimulate deeper cognitive engagement and enhance mathematical understanding (Baki, 2008).

5. Promotion of Communication: Encouraging communication among students through activities is crucial. It facilitates peer learning, where students can articulate their understanding, ask questions, and collaboratively build upon their collective knowledge. This communication helps in reinforcing and refining their understanding of concepts (Suzuki & Harnisch, 1995).

In essence, these characteristics emphasize that activities in mathematics education should not be mere tasks but rich experiences that promote deeper understanding, application, and collaboration among students.

Certainly, the overarching aim of activity-based teaching in mathematics education is to deepen students' mathematical understanding. However, as indicated by the aforementioned definitions, there is a tendency in some studies to side-line the mathematical essence when prioritizing efficiency. At times, the inherent mathematical value within these activities is not given its due emphasis. Given the substantial demands these activities place on teachers, and the roles assigned to students, it is essential that the anticipated mathematical advancement from this approach should be worthy of the invested effort. Certain research touching upon this nuance have contextualized it within terms like “mathematically rich tasks” or “rich mathematical tasks”. This line of discussion is crucial for it shed some important light on the often-overlooked mathematical dimension in the discussions on activity-based teaching. As such, a more comprehensive look into the concept of rich mathematical tasks seems both relevant and necessary.

2.2. Rich Mathematical Tasks

Mathematical activities aim to stimulate mathematical growth upon completion (Liljedahl et al., 2007). This mathematical growth can manifest in various ways. It includes generating mathematical ideas, applying mathematical knowledge, having a positive disposition towards mathematics, doing mathematics using various tools (symbols, diagrams, tables, graphs, and models, etc.), and solving a problem in a real-life context (Fitriati & Novita, 2018). Fitriati and Novita provide an example of a rich task, as depicted in Figure 2.2.1.

Febi and her family are in Frans Studio Bandung. They plan to enjoy the game in the playgrounds. The beside picture is one of the leaflets informing some games and the requirements.
If the information of Febi and her family’s weight and height are shown in the table below:

No	Subject	Weight	Height
1	Febi	55 kg	149 cm
2	Brother	24 kg	126 cm
3	Father	70 kg	173 cm
4	Mother	51 kg	162 cm

Febi dan keluarga sedang berada di Trans Studi Bandung. Mereka berencana ingin menikmati permainan pada wahana bermain tersebut. Gambar di samping adalah salah satu selebaran yang memuat informasi beberapa permainan dan persyaratannya.

Jika informasi tinggi dan berat badan Febi dan keluarga seperti yang ditunjukkan pada Tabel di bawah

No	Subjck	Berat Badan	Tinggi Badan
1	Febi	55 kg	149 cm
2	Adik	24 kg	126 cm
3	Ayah	70 kg	173 cm
4	Ibu	51 kg	162 cm

Maka, dapatkah kamu membantu Febi dan keluarganya untuk menentukan permainan apa saja yang dapat dimainkan mereka?

Then, can you assist Feby and her family in determining the games that they can play?

Mini Bumper Boats
Requirements:
The maximum height is 130 cm
The minimum height is 100 cm
The maximum weight ise 25 kg

Jump Around
Requirements:
The maximum height is 130 cm
The minimum weight is 50 kg

Giant Swing
Requirements:
The maximum height is 130 cm
The minimum weight is 90 kg

Figure 2.2.1. Example of rich task

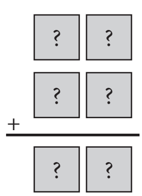
The task illustrated in Figure 2.2.1 offers a practical, real-world scenario. In this context, Febi and his family visit a playground and are faced with the task of selecting appropriate games for each family member based on the options provided in a brochure. According to Fitriati and Novita (2018), this qualifies as a "rich" task because it situates students in a real-world problem-solving context, demanding active engagement. In undertaking this rich task, students are required to form meaningful connections between the data presented in the table and the actual needs of each family member, all set within a context they encounter in daily life.

In discussing the essential features of rich tasks within the realm of mathematics education, Griffin (2009) posits that such tasks should meet several criteria. Specifically, they should provide students the opportunity to make choices, avoid complex narratives, eliminate the necessity for rote memorization, and refrain from overwhelming students with an abundance of symbols. Additionally, these tasks should be designed to foster an environment where students can meaningfully engage with mathematical concepts. This focus on cognitive engagement and active involvement in mathematical problem-solving is noteworthy. In accord with Griffin's criteria, Martin (2003) offers an example of a rich task, which is depicted in Figure 2.2.2.

DOMINOES

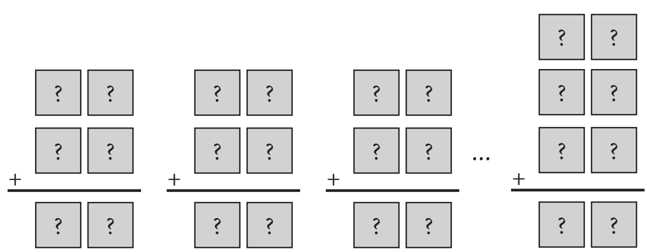
Materials: A full set of twenty-eight [28] dominoes

1. Can you place three [3] dominoes so that they make a correct addition?



2. Can you find a combination which involves carrying?

   Can you arrange all twenty-eight [28] dominoes into nine sums which all show a correct addition?



This can be done without carrying, OR it can be done with some carrying

Figure 2.2.2. The domino activity

In Figure 2.2.2, the "domino activity" involves a mathematically rich task, frequently prompting students for inquiries. The instruction, "Can you place three [3] dominoes so that they make a correct addition?" not only offers students a decision-making opportunity but also empowers them to actively engage with mathematical thinking. This task is tailored to cater to a broad spectrum of student abilities, yet remains sufficiently challenging to stimulate those with advanced cognitive skills. Martin (2003) points out that the task avoids dense narratives and eschews the need for rote memorization. Moreover, it paves the way for a myriad of problem-solving strategies, including but not limited to approaches like guess-check-apply and working backward to discern components from their sum.

Goos et al. (2013) offer an insightful commentary on arithmetic tasks, arguing that genuine arithmetic proficiency extends beyond merely knowing and employing effective methodologies. Such expertise also encompasses the ability to assess the plausibility of results and discern the aptness of mathematical reasoning in different contexts. This perspective underscores the necessity for arithmetic tasks to nurture a critical mindset in learners. Drawing from this understanding, Goos et al. (2013) extract the following characteristics that rich arithmetic tasks should ideally embody:

- They should necessitate the application of mathematical knowledge.
- They ought to foster positive dispositions in students, such as confidence, initiative, and a flexible, adaptive approach to applying mathematical insights.
- They should create avenues for the utilization of various tools.
- They must be instrumental in developing students critical thinking skills.
- They should facilitate a pedagogical approach that promotes exploration and inquiry.

Piggott (2012) offers insights into the attributes of rich mathematical tasks, shedding light on the facets that make these activities particularly engaging and effective. According to Piggott, tasks that captivate students immediately upon introduction and present multiple layers of challenges are invaluable. Such tasks actively encourage students to undertake decisions, venture guesses, formulate and assess hypotheses, and delve into proving and elucidating their reasoning. These activities further prompt reflection, interpretation, and the synthesis of rich mathematical experiences. Complementing this perspective, Mason (2020) emphasizes that the essence of rich mathematical tasks lies in their capacity to stimulate active participation, foster and diversify cognitive patterns, and guide learners towards self-driven generalizations.

Researchers, delving into the intricacies of rich mathematical tasks, often describe them in the context of specific mathematical processes they serve. For instance, Suzuki and Harnisch (1995) posit that tasks can be deemed rich in mathematical content when they are centred around certain key elements. These include:

– The ability to model real-world scenarios,
– The incorporation of multiple solution strategies and representations,
– The facilitation of connections between diverse mathematical concepts to derive a solution,
– Encouraging verbal articulation of solution strategies,
– And discerning the differences between a heuristic solution and a purely mathematical one.

By encompassing these facets, tasks can immerse students in a deeper and more holistic mathematical experience. Griffin (2009) offers an approach that resonates with the views of Suzuki and Harnisch concerning the conceptualization of rich mathematical tasks. Griffin outlines five distinctive types of tasks which can underpin the creation of rich mathematical activities. These are:

– Classification of mathematical objects,
– Interpretation of multiple representations,
– Evaluation of mathematical expressions,
– Problem establishment and
– Analysing the rationales and solutions.

Griffin (2009) underscores the crucial point that the mere introduction of rich mathematical tasks is not sufficient; they must be integrated with the right pedagogical approaches to truly foster valuable mathematical development. Griffin delineates eight fundamental characteristics that should be embodied in such an effective pedagogical approach:

– Grounding instruction on the existing knowledge base of students,
– Identifying and addressing misconceptions,
– Facilitating thoughtful and productive inquiries,
– Promoting collaborative work within small groups,

- Placing emphasis on methodologies rather than mere end-results,
- Deploying tasks that nurture cooperative learning,
- Drawing connections among various mathematical ideas, and
- Ensuring effective incorporation of technology into instruction.

These characteristics, as articulated by Griffin, shape a comprehensive instructional framework which, when combined with rich tasks, can greatly enhance mathematical understanding. Echoing this perspective, several scholars, including Glover (2016), Foster (2017), and Fitriati and Novita (2018), have established that rich tasks serve as pivotal tools to stimulate higher-order thinking skills. Complementing this view, Piggott (2012) has illuminated how these tasks inspire students to venture into devising original solutions and innovative approaches. Furthermore, the utility of such tasks in educational settings has been highlighted—they not only aid students in the practical application of their mathematical knowledge but also refine their intricate problem-solving abilities (Ferguson, 2009; Margolinas, 2013). Further emphasizing the profound impact of these tasks, Fitriati et al. (2020) identified that reflective thinking, a key component of higher-order cognitive processes, can be effectively nurtured through them.

In conclusion, there are four important points that the literature review on rich mathematical tasks points out. First, it is necessary for rich mathematical tasks that students can express their thoughts, make comments, provide reasons, be encouraged for analysis and evaluation, and apply their mathematical knowledge to new and different contexts. Second, targeted mathematical development opportunities emerge if rich mathematical tasks are implemented using appropriate pedagogical approaches. Third, rich tasks can support students' cognitive, affective, and intellectual development. Finally, it is important that the mathematics embedded within these tasks possesses inherent value for the learners, ensuring that the resulting mathematical enrichment is both meaningful and substantial.

2.3. Components of Design and Implementation of Activities

Activity-based instruction plays a significant role in shaping students' mathematical development. However, the extent of this impact is intricately tied to the way in which they are designed and executed. As pointed out by Watson et al. (2013), the constituent elements that contribute to a high-quality activity often receive inadequate attention in research. Surprisingly, there is a notable dearth of comprehensive studies addressing the holistic design and execution of activities while focusing on the attributes that underlie these processes. Yet, numerous investigations have been conducted to probe the specific attributes inherent to the design and execution of activities. This section of the book delves into these findings and elucidates the key factors influencing the quality of both activity design and implementation.

Purpose of the Activity: Every activity is designed with a specific intent and a desired outcome in mind. To realize this outcome, students engage in tasks and produce outputs as part of the activity. The tasks undertaken and the outputs produced should align with and ultimately fulfil the intended purpose of the activity. Understanding the goal of the activity or the intended product provides essential guidance for its execution (Henningsen & Stein, 1997). Decisions related to the activity's execution, such as student engagement strategies, material functionality, and concluding procedures, should always align with its primary objective. However, pinpointing the exact purpose of an activity can be challenging. For instance, when Güzel et al. (2020) provided teachers with activities and prompted them to state their objectives, diverse goals were ascribed to the same activity. A single activity can cater to multiple objectives or be interpreted differently by various teachers. In such scenarios, ensuring the teacher has a precise understanding of the activity's purpose becomes crucial for the successful implementation of the activity.

Instructions: Activities are comprised of directives and sequential steps aimed at achieving a specified outcome. These directives and steps are collectively termed as 'instructions'. It is imperative that instructions are tailored to fulfil the activity's objective. They should be comprehensive, devoid of superfluous details, and clearly articulated, ensuring clarity and linguistic accuracy (Coles & Brown, 2016). This clarity ensures that students face challenges solely aligned with the activity's intended purpose. Barrett and Battista (2014) emphasize that educational goals set by teachers often necessitate significant strides, which can be overwhelming for students to achieve in a singular effort. Hence, instructions should facilitate the execution of the activity by segmenting it into manageable chunks. To illustrate the significance of well-structured instructions, a sample activity from Üstündağ Pektaş (2018, p.90) is useful as presented in Figure 2.3.1.

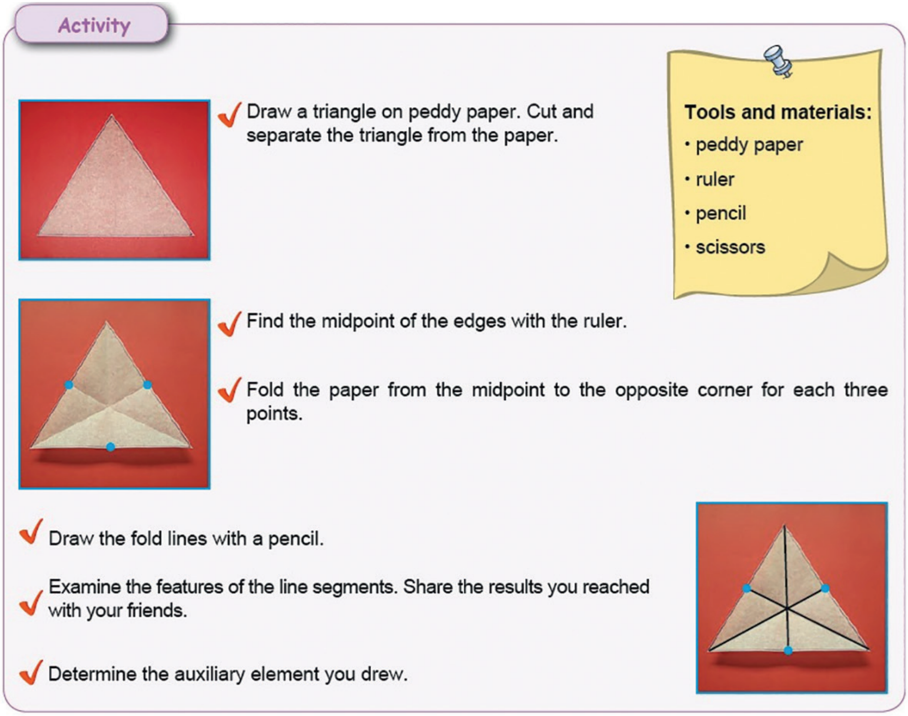


Figure 2.3.1. Median construction activity in the triangle (Üstündağ Pektaş, 2018, p.90).

Güzel (2020) observed that when a teacher administered the activity depicted in Figure 2.3.1, students struggled to cut out triangles and identify the auxiliary elements of the triangles based on the line segments they drew. The initial instruction required students to cut triangles from peddy paper. However, due to their unfamiliarity with materials like peddy paper and scissors, students encountered difficulties in drawing and cutting. Güzel emphasized the need for additional guidance in the instructions to address this issue, highlighting the significance of clear instructions for the success of activity-based instruction.

Use of Materials: The selection of appropriate materials is a crucial element in the design and execution of activities. These materials can range from specialized educational tools like algebra tiles, counting stamps, and tangrams, to software solutions such as dynamic geometry or algebra programs and smartphone applications. Everyday objects, or 'realia' (e.g., picture frames, oranges, pencils), can also serve as effective learning mediators. Cameron and Bennett (2010) have emphasized that material-supported mathematics education enhances perception, learning, and retention. Additionally, well-chosen materials can enable students to overcome both instructional and epistemological barriers (Brousseau, 1998). During the activity design phase, it is vital to carefully consider the materials that will be utilized, how they will be used, and their respective benefits and limitations. If not thoughtfully selected, materials can inadvertently introduce new challenges, detracting from the activity's intended purpose. For instance, the activity in Figure 2.3.2, cited from a MEB study (2018, p.85), serves as a relevant example.

Let's Try This

Required Materials : Cardboard, pencil, ruler, scissors or utility knife

- Draw at least twenty independent rectangles on cardboard with a width of 3 cm and a length of 4 cm.
- Cut the rectangles you have drawn along the edges.
- Create a quadrilateral region by using the minimum number of these rectangular cardboards.
- Find the length of one side of the square you created.
- Discuss the relationship between the length of one side of the square and the multiples of 3 and 4.
- Do the same for rectangles with a width of 4 cm and a length of 6 cm.

Figure 2.3.2. Finding the common multiple of two natural numbers (MEB, 2018, p.85)

In the activity depicted in Figure 2.3.2, students are tasked with creating congruent rectangles and subsequently forming a square from these rectangles. This requires them to draw and cut at least twenty rectangles. However, ensuring precision in the cutting process is a notable challenge. The tasks of drawing and cutting, apart from being time-consuming, can also introduce confusion. The extent to which these processes align with the primary objective of the activity remains questionable. Furthermore, the use of tools like utility knives presents safety concerns. Hence, when selecting tools and materials, it is crucial to weigh their potential to both facilitate cognitive processes and introduce complications. Above all, the chosen resources should align seamlessly with the activity's intended goals.

Inclusivity: Activities should be designed in a way that allows full participation of all target students. Ensuring inclusivity means that every student should have equal access to the classroom where the activity takes place (Cornwall & Graham-Matheson, 2012). It is crucial that activities promote inclusivity, ensuring that no subset of students is disadvantaged. To achieve this, activities should be grounded in contexts familiar to the students. Moreover, if an activity introduces specific jargon or terms, clear explanations should be provided to ensure understanding. For instance, defining the term “pentomino” in the activity presented in Figure 2.3.3 by Liljedahl et al. (2007, p.242) underscores the importance of clarity to foster inclusiveness.

A pentomino is a shape that is created by the joining of five squares such that every square touches at least one other square along a full edge. There are 12 such shapes, named for the letters they most closely resemble.

Now consider a 100’s chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

If a pentomino is placed somewhere on a 100’s chart will the sum of the numbers it covers be divisible by 5? If not, what will the remainder be?

Figure 2.3.3. Pentomino activity (Liljedahl et al., 2007, p.242)

Upon closely examining the structure of the activity in Figure 2.3.3, it becomes evident that the initial tasks (such as placing the pentomino and summing the numbers) are tasks that nearly all students can undertake. This highlights an essential strategy for inclusivity: setting a low entry threshold. Nonetheless, as pointed out by Liljedahl et al. (2007), the subsequent generalizations within the activity demand a robust mathematical foundation. This offers students who excel an opportunity to delve into and discover new mathematical attributes. Thus, the dual approach of having a low entry threshold while catering to students across various achievement spectrums significantly enhances the inclusive nature of activities.

Student Roles and Responsibilities: The roles and responsibilities allocated to students within an activity can substantially impact the effectiveness of that activity (Watson & Mason, 2007). An optimally designed activity should empower students to take charge of their own learning, formulate hypotheses based on presented problems, and rigorously test these hypotheses. In essence, the design should encourage students to take intellectual risks (Bransford & Donovan, 2005). Therefore, it is vital to establish clear guidelines concerning the roles students will play and the responsibilities they will undertake within the activity. Specifically, the instructions should unambiguously indicate to whom they are addressed, delineate the mode of collaboration during group activities, and specify the responsibilities of each group member (Özmantar & Bingölbali, 2009).

Readiness: For an activity to be effective, it should align with students' prior knowledge, ensuring that the language, terminology, and instructions are appropriate for the students' level. Furthermore, it should take into account students' familiarity with the selected context and their experience with the materials being used (Swan, 2007). Prior knowledge serves as a cornerstone in selecting suitable activities for successful engagement. However, readiness might not always be evident during the design and execution of activities. As highlighted by Güzel (2020), challenges related to readiness can often present complexities that are not easily addressed through teacher interventions. While the concept of readiness is undeniably crucial, the debates surrounding it tend to be largely theoretical, due to its elusive nature and the fact that interventions are not always effective. In practical classroom settings, readiness becomes an attribute that teachers need to intuitively factor in, grounded in their understanding of their students.

Attention Management: Effective attention management during activity implementation entails two key components: (1) initially engaging students to participate in the activity and (2) directing their focus towards the intended learning outcomes (Özmantar & Bingölbali, 2009). Research indicates that student motivation increases when activities are intrinsically interesting (Ainley et al., 2006; Nyman, 2016). However, the manner in which the teacher introduces the activity is equally critical for garnering student participation. Preliminary briefings that inform students about the activity, stimulate their curiosity about the expected outcomes, and provide insight into the activity's relevance are essential for engagement (Ainley et al., 2006). Furthermore, the teacher's emphasis on salient aspects during the execution of the activity significantly influences students' comprehension and realization of the targeted learning objectives (Choy, 2016).

Time Management: Given the unique dynamics of each classroom or group, effective time management is paramount for the successful implementation of activities. Inadequate time allocation can hinder students from achieving the intended learning outcomes, while excessive time may lead to unproductive digressions (Stein, Grover, & Henningsen, 1996). A critical facet of time management involves judiciously partitioning the available time across various stages of the activity. Activities inherently comprise multiple phases—ranging from individual or group-based independent work to product exchanges, class presentations, and reflective discussions (Guzel, 2020). These stages are not uniform across all activities and may vary in number and nature. Consequently, it is crucial to allocate time in a manner commensurate with the significance of each stage to the overall implementation, ensuring that each phase receives the attention it warrants.

Teacher Intervention: Teacher intervention encompasses the guidance and support provided to students during an activity's implementation. This support can manifest in various forms, including the teacher's explanations, answers to questions, focal points of emphasis, and even non-verbal cues like gestures and facial expressions. Teachers have the capacity to process and contextualize student inputs (Doerr, 2006) and to interconnect responses in a mathematical framework (Swan, 2007). The crux, however, is selecting an intervention form that aligns with and advances the desired learning objectives.

Henningsen and Stein (1997) noted instances where, in response to student challenges, teachers would either directly provide the expected answer or diminish cognitive demand by guiding students too closely towards the solution. Optimally, teacher interventions should bolster students' capacities to persist with and navigate the activity, aiding them in reaching the intended outcomes. Furthermore, timely and suitable interventions are vital for students who face hurdles during the activity or who inadvertently hinder the flow of the session (Güzel et al, 2021). Viewed in this light, the nature of teacher intervention is multi-faceted, with judicious intervention being pivotal for a successful activity execution.

Concluding the Activity: Wrapping up an activity necessitates synthesizing all the work undertaken and clearly presenting the targeted outcome or product. When drawing an activity to its close, teachers have an array of strategies at their disposal: they might articulate the intended result directly, encourage students to unearth the mathematical conclusion, elucidate by highlighting student errors, or assign tasks for students to contemplate without explicit guidance (Özmantar & Bingölbali, 2009).

Watson (2008) posits that the culmination of an activity is a pivotal juncture, with its direction largely influenced by the teacher's pedagogical stance. The essence of a successful conclusion lies in ensuring that all efforts during the activity coalesce into a coherent understanding, revealing the inherent mathematical principles. The teacher's chosen strategy, therefore, should align with and amplify this overarching goal.

2.4. Evaluation Approaches to Activity Design and Implementation Processes

While there are limited studies that delve into the assessment approaches related to designing and executing activities in mathematics education, a few have emphasized the importance of a cyclical evaluation process rooted in practical application. A notable contribution to this field is the study by Liljedahl et al. (2007). The researchers assert that the essence of quality in both designing and executing activities lies in a repetitive, cyclical evaluation process. This process can be delineated into stages: predictive analysis, practical trial, reflective analysis, and subsequent adjustment. In this model, a high-quality mathematical activity undergoes pre-implementation evaluation, actual implementation, post-implementation reflection, and necessary refinements.

While this repetitive evaluation model undoubtedly advocates the enhancement of instructional activities' quality, it presents the determinants of activity quality as being process-centric rather than component-based. Such an approach underscores the importance of teacher reflection on the chosen activities. However, it leaves certain gaps. Specifically, it does not pinpoint the key components that should be the focal point for creating and deploying a high-quality activity. Another significant limitation is the lack of clarity on guiding principles for the design and execution phases. The study does not elaborate on a structured framework with clear indicators or rating criteria, which could have provided more tangible benchmarks for assessment.

An alternative evaluation approach is proposed by Güzel and colleagues, as detailed in their studies (Güzel, 2020; Güzel et al. 2021). This approach integrates both the process model and its associated components in the appraisal of mathematical activities. Within this framework, assessments are conducted iteratively, encompassing both the design and the implementation phases. Initial evaluations, predicated upon the objectives set during the design stage, are revisited and expanded upon following the activity's execution and conclusion. Consequently, this model conceptualizes the process of activity design, implementation, and evaluation as a continuous cycle. Importantly, post-implementation assessments in this model serve as catalysts for revisions to both design and implementation in future iterations. However, the model is not without its shortcomings. Specifically, it lacks clear guidelines or indicators to steer the evaluation process, thereby raising questions about how activities should be assessed in alignment with the proposed principles.

A salient feature common to the approaches discussed is the emphasis on continuous evaluation for optimal activity design and successful implementation. It appears crucial to persistently assess even those activities perceived to run flawlessly, given that identical activities may yield different outcomes across various groups (Revina & Leung, 2019). Factors such as distinct sociocultural contexts, socio-mathematical backgrounds, or readiness levels of different groups can profoundly influence the execution of an activity (Revina & Leung, 2018). Thus, enhancing the quality of both the design and the implementation is contingent upon teachers' commitment to a relentless evaluation process.

To ascertain the quality of activities and relevant practices, it is crucial to assess and consequently provide feedback, which in turn informs teachers about their pedagogical strategies and the learning process. Sadler (1989) postulates that feedback bridges the discrepancy between actual performance and the desired standard. Molloy and Bound (2012) argue that feedback addresses three primary inquiries: "Where am I headed?", "How am I performing?", and "What steps should I take next?". Feedback, in this regard, offers a reflective perspective, illuminating the appearance of one's performance (Molloy & Bound, ibid.). Feedback, more than just supplying information, plays a pivotal role in driving positive performance changes. It serves as a catalyst, encouraging both reflection and action towards improved performance and understanding. Such feedback grants individuals insights into their performance, fostering the ability to adjust, enhance, and rectify potential shortcomings.

However, the cornerstone here is the provision of pertinent, prompt, and effective feedback. When seamlessly integrated into the performance, such feedback has the potential to elevate the design and effectiveness of activities. Evaluation serves as the foundation for generating this feedback. But, as gleaned from preceding discussions, the tools available for teachers to facilitate this evaluation are somewhat scarce and often only offer broad, generalized insights that lack practical value. What is genuinely needed is an evaluative framework that equips teachers with actionable feedback. This entails a structure built upon discernible performance metrics, established criteria, and tangible indicators to inform both the design and execution phases of activities. This book seeks to address this gap, presenting readers with a tool for activity evaluation and feedback that promises both practicality and efficacy.

CHAPTER 3

DEVELOPMENT OF THE MATHEMATICAL ACTIVITY EVALUATION AND FEEDBACK TOOL

The "Framework for Mathematical Activities" (FfMA) was crafted using the principles of design-based research (DBR). DBR is characterized by its collaborative nature, involving both researchers and practitioners, with the aim of generating outcomes that have both practical applicability and theoretical significance. While there are various models elucidating the intricacies of the DBR approach, one of the most encompassing and widely-accepted frameworks was proposed by Wademan (2005 cf. Keser Özmantar, 2018).

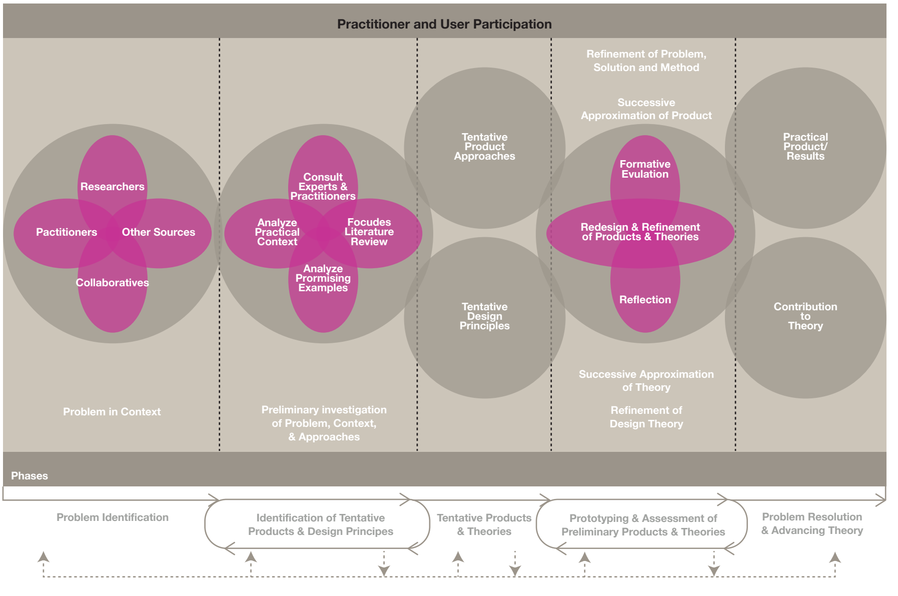


Figure 3.1. Design-based research process and stages (Wademan, 2005)

As illustrated in Figure 3.1, at the heart of DBR lies the collaborative endeavour between researchers and practitioners to address real-world challenges. Key aspects of DBR involve formulating principles aimed at resolving the identified problem and subsequently testing their efficacy within real-world settings. The overarching objective is to continually refine and enhance these principles based on empirical findings. Ultimately, the aim is to either extrapolate these principles into a theoretical framework or to derive a tangible product informed by these refined principles.

During the development of the FfMA, the project closely adhered to the five-stage model recommended by Wademan (2005). Initially, efforts were concentrated on identifying and defining the specific problem that the framework aimed to address. Once the issue was clearly outlined, the second stage involved delineating the dimensions of the FfMA and establishing its fundamental design principles. With these foundations laid, the third attended to the creation of an initial draft version of the FfMA for the purpose of pilot testing. The fourth stage was characterized by real-world trials of this draft, followed by a series of iterative refinements based on the data and feedback gathered. Ultimately, the fifth and final stage culminated in the formation of the definitive version of the FfMA, which was then documented to highlight both its practical value and theoretical contributions. This chapter provides an in-depth look at each of these sequential steps that were instrumental in shaping the FfMA.

3.1. Stage One: Defining the Problem

The first stage of the DBR approach focuses on expressing the problem in all its dimensions and in its real environment carefully. At this stage, a comprehensive literature review was conducted, the academics who have studied the use of activity in mathematics education were consulted, teachers with activity-based teaching experience were collaborated, and their classroom practices were observed. One online and one face-to-face workshop was organized with academics working on mathematical activities. Five academics contributed to each workshop. In workshops, the participants shared and discussed the definition of mathematical activity, the qualities of successful activities, the selection of activities, the challenges and problems encountered in the implementation, and the factors that influence the success of activity-based instruction.

One online and one face-to-face workshop were conducted with secondary school mathematics teachers who incorporate mathematical activities into their teaching. The online workshop was attended by five teachers, while the face-to-face workshop had seven participants. During these workshops, the teachers discussed various topics, including the availability and quality of mathematical activities, criteria for their selection and adaptation, challenges faced during implementation, and factors influencing the success of implementing mathematical activities. Supplementing these workshops, we have also performed in-class observations of five teachers who employed activities in their teaching. These observations allowed for a direct examination of the practical challenges teachers encounter when implementing mathematical activities in actual settings.

Through collaborations with participants and direct classroom observations, our research sought to uncover both theoretical and practical issues related to mathematical activity implementation, thereby delineating the scope of the problem. Analyzing the data collected in the first stage led to an operational definition of the concept of mathematical activity, which we will share later. Additionally, we conducted exploratory studies to identify the key characteristics that should be taken into account for designing and implementing high-quality mathematical activities.

3.2. Stage Two: Identifying the Dimensions and Indicators of F*f*MA

In the second stage of developing the FfMA, we focused on identifying the key dimensions that contribute to effective activity-based mathematics instruction. We also worked on constructing a model to elucidate the relationships among these identified dimensions. Simultaneously, we designated the essential components that govern both the design and implementation phases of mathematical activities. To facilitate a nuanced evaluation of these components, we established specific, observable indicators. This stage of development was significantly informed by a thorough analysis of previously collected data and an extensive review of existing academic literature.

To ensure the content validity of the FfMA and to scrutinize its various components and indicators, we sought the opinions and evaluations of both academics and practicing teachers. To this end, two distinct focus group interviews were organized. The first comprised seven academics who were involved research on mathematical activities, while the second included seven teachers experienced in implementing activities in their classrooms. At the onset of each interview session, participants were introduced to the guiding model behind the development of FfMA through a comprehensive presentation. This presentation detailed the quality-determining components of activity design and implementation, along with the corresponding indicators. Subsequent to the presentation, participants engaged in a critical evaluation of the proposed model, its components, and indicators.

Their input helped us identify the strengths and weaknesses of the draft model, as well as areas requiring further refinement. In addition to the focus group feedback, we conducted a critical analysis of video recordings showcasing the teaching practices of five teachers in actual classroom settings. Furthermore, we analysed videos of activity implementations from real classrooms that were publicly accessible on various platforms, such as the National Council of Teachers of Mathematics and Inside Mathematics. These empirical investigations enriched our understanding of the practical dimensions and components of FfMA, offering invaluable insights for its further development.

In the second stage of the FfMA development process, our objective was hence multi-faceted. We aimed to identify the dimensions crucial to successful activity-based mathematics instruction, and to elucidate the relationships among these dimensions. Additionally, we focused on specifying the components and indicators that significantly impact the quality of both the activity design and its implementation. The data collected and analysed during this stage were instrumental in crafting the initial version of the FfMA.

3.3. Stage Three: Creating the First Version of F*f*MA

The F*f*MA was devised as an analytical rubric. Such a rubric is employed to evaluate each individual component of a given performance. Employing an analytical rubric facilitates a nuanced understanding of the strengths and weaknesses of a performance or task by focusing on its individual components. Each component undergoes separate evaluation. Dissecting the overall judgment regarding a performance into more manageable segments fosters increased clarity and aims to enhance objectivity in scoring, according to Sadler (1989). Consequently, this approach augments the reliability of the assessment outcomes.

In the process of crafting the analytical rubric, distinct attributes that determine the quality of each performance component were selected and detailed. To develop the initial version of FfMA, a thorough assessment of data from the first and second stages was executed. This comprehensive analysis led to the structure of activity design and implementation as two distinct processes. Key features of effective activity design and implementation were categorized and associated with specific components. From these identified features, indicators that define the quality of each component were crafted. During this phase, care was taken to ensure that these indicators were clear, distinct, observable, and interconnected, avoiding redundancy across different components.

For each component, distinct indicators were devised to determine the spectrum of performance quality, ranging from the lowest to the highest. This spectrum was delineated into four distinct scoring levels: very low (0 points), low (1 point), moderate (2 points), and high (3 points). The initial step in this scoring process was to articulate the best and worst performances for each component. By juxtaposing these extremities, criteria for intermediate levels were formulated, leading to the crafting of performance definitions for moderate scores. This provided a clear foundation for the scoring criteria.

In adherence to this structured process, the initial version of FfMA was developed, outlining the components, indicators, performance ratings, and evaluation criteria pertinent to both activity design and implementation.

3.4. Stage Four: Successive Efforts for Improvement

In the fourth stage of the FfMA's development, we undertook rigorous refinement and enhancement processes. This phase was pivotal in the prototype evolution of the FfMA tool, spanning two academic years and unfolding in two distinct cycles. During each cycle, we engaged with different sets of middle school mathematics teachers.

In the initial cycle, participants put the initial version of the FfMA to the test in actual settings, providing reflective assessments based on their experiences. Informed by feedback from this cycle, the FfMA underwent revisions and improvements. Subsequently, in the second cycle, a fresh cohort of teachers deployed the enhanced FfMA in practical settings. Post-implementation, these teachers were encouraged to offer their reflective insights.

By juxtaposing feedback from both cycles and weighing the insights provided by the teachers, we performed comprehensive analyses. This review process paved the way for the formulation of the definitive version of the FfMA. This approach aimed to ensure that the FfMA was both robust and reflective of the complexities inherent in the realities of the actual settings. The details of the both cycles, briefly described above, are explained in greater detail below.

Improvement studies– First cycle: During the first cycle of improvement studies, 23 middle school mathematics teachers participated. These teachers underwent a 10-hour training session. The training began with an introduction to the theoretical foundations of activity-based teaching. This was followed by an introduction to the initial version of FfMA, along with guidelines on its application. To elucidate its usage, examples were presented using selected activity scripts and video recordings from actual classroom sessions. For the initial evaluation, the research team provided the participants with an activity script, highlighting how teachers can select and adapt activities. For the subsequent assessment, participants were asked to implement an activity in their own classrooms and subsequently reflecting upon the process. The researchers also showcased two activity implementation videos for the third and fourth evaluations. These exercises equipped teachers with the skills to critically evaluate activity implementations using the FfMA as a guiding tool.

Following the training, the teachers were instructed to select and implement five activities in their respective classrooms, utilizing FfMA as a guide for both the selection and the implementation processes. Subsequently, the teachers provided feedback on the functionality and utility of FfMA, specifically in terms of aiding activity selection/adaptation and offering insights into their practices.

The primary aim of the first cycle was to ensure that users comprehensively understood the FfMA, thereby enabling more effective contributions to its refinement. To achieve this objective, we considered the teachers' critical assessments of FfMA's strengths and weaknesses, which were informed by their practical experiences over a specified period.

During this phase, an in-depth analysis was conducted by closely observing three teachers selected from among the participants. These teachers were specifically chosen because they had not only implemented activities in their classrooms but had also engaged in improvement studies by critically evaluating these activities using the FfMA framework. Their processes of activity selection, adaptation, and implementation were monitored by the researchers in authentic classroom settings. In addition to observing their practices, researchers provided these teachers with structured feedback. Through collaborative assessments, a nuanced understanding of FfMA's role in shaping instructional practices, as well as its functionality, strengths, and areas needing improvement, was achieved.

Improvement studies – Second cycle: In the first cycle, the FfMA framework underwent revisions based on the analysis of data gathered from participant interactions and usage. The improved version of FfMA was then rolled out in the second cycle, subjected to scrutiny by a new group of participants comprised of 12 middle school mathematics teachers. Consistent with the first cycle, these participants were also offered training sessions that covered identical content within first cycle. Activities that had been executed with the initial group of teachers were replicated with this new cohort to ensure comparability.

During the second cycle, the teachers were introduced to the updated FfMA framework and were encouraged to employ it in their classroom practices.

Specifically, they were asked to select, implement, and critically evaluate activities using the FfMA framework. Following both the training sessions and their in-class implementations, feedback was gathered from the teachers regarding the framework's strengths, weaknesses, and overall utility. This iterative approach helped in further refining the FfMA based on real-world applications and feedback.

Upon concluding the second cycle, the participants were grouped into two sets of six, and focus group interviews were conducted. During these sessions, feedback concerning various aspects of the FfMA—including its components, indicators, scoring criteria, as well as the clarity and utility of the scores—was documented. Additionally, these sessions aimed to document any challenges or issues the participants encountered while utilizing the FfMA. The insights gained from this cycle were instrumental in finalizing the FfMA.

3.5. Stage Five: Finalising the F*f*MA

In the fifth and final stage, the focus was on generating the ultimate version of the FfMA and drawing theoretical conclusions. To create this final iteration, we thoroughly analyzed the feedback provided by teachers who participated in the second cycle of the improvement studies. Any issues highlighted by the participants led to subsequent refinements of the FfMA. We also conducted a comparative review of participant feedback across both cycles to ascertain if issues identified in the initial cycle persisted. Guided by these comprehensive analyses, the final adjustments to the FfMA were performed.

The data collected from participants throughout the improvement cycles offered valuable insights into various aspects of the FfMA. These included its effectiveness in differentiating between performance levels, the clarity of its scoring criteria, the consistency of scoring, and the overall utility of the tool. Such information not only facilitated the creation of the final version of the FfMA but also provided a basis for theoretical inferences concerning its applicability and effectiveness. As a result of this development process, the FfMA has been confirmed as a viable tool for actual classroom settings.

3.6. Validity, Reliability and Usability of the F*f*MA

An effective assessment tool must possess three fundamental attributes: reliability, validity, and usability (Buchheit et al., 2010). Reliability pertains to the consistency of the results generated by the tool, ensuring that scores are dependable across different instances. Validity involves an appraisal of the tool's effectiveness in fulfilling its intended purpose. Usability, meanwhile, concentrates on the tool's ease of use and practicality in implementation.

The developmental studies conducted for FfMA were designed to rigorously address each of these critical aspects. Through multiple cycles of participant feedback, iterative refinements, and comparative analyses, the FfMA has been substantiated as a reliable, valid, and user-friendly tool, apt for application in real-world educational settings.

The validity of the FfMA can be broken down into three primary dimensions: content, construct, and criterion validity. As delineated by Jonsson and Svingby (2007) as well as Moskal and Leydens (2000):

- Content validity evaluates if the assessment tool:
  - Accurately identifies relevant content without including any irrelevant elements.
  - Adequately addresses all aspects of the intended content.
  - Ensures there are no ambiguities or unspecified areas in the content that is set to be evaluated by the tool.
- Construct validity determines:
  - If the assessment captures all essential aspects of the intended construct with its scoring criteria.
  - The extent to which all scoring criteria align with and are pertinent to the relevant construct they intend to measure.
- Criterion validity addresses:
  - How effectively the scoring criteria depict aspects of relevant performance or potential for future success.
  - Any possible facets of associated performance that might not be encapsulated in the scoring criteria.

In essence, these dimensions of validity work together to ensure that an assessment tool, such as the FfMA, effectively, comprehensively, and accurately measures what it sets out to evaluate. Considering the processes involved in the development of the FfMA, it is evident that significant measures were taken to ensure its content, construct, and criterion validity. Below is a brief description of the works undertaken for each validity category.

Content Validity:

- To ensure full coverage of the designed content, academic experts, practicing teachers and a comprehensive review of relevant literature were consulted to identify the dimensions and components of FfMA.
- Real-world classroom observations ensured that no irrelevant or undefined content areas were included in FfMA.

Construct Validity:

- Teachers were trained on the first version of FfMA to ensure all scoring criteria are related to the relevant construct of classroom activity assessment.
- Based on real-world application and feedback from teachers and experts, all important aspects of the intended construct were evaluated. Areas for improvement were identified and handled for successive refinement of the tool.

Criterion Validity:

- Teachers used FfMA in real classroom settings, providing practical feedback that was vital in assessing how well the scoring criteria reflected actual classroom performance.
- Focus group interviews provided insights into how well the FfMA's scoring criteria aligned with teachers' perceptions of successful outcomes, thus examining aspects of relevant performance.

The efforts documented thus far have been also instrumental in ensuring the reliability of FfMA. For any tool to be deemed reliable, it should yield consistent results across successive measurements (Baykul, 2000). Moreover, these measurements should be a true reflection of the content, demonstrating repeatability when applied to the same individuals under comparable conditions (Crocker & Algina, 1986). Reliability in assessments made using such tools encompasses sensitivity, stability, and consistency (Baykul, 2000). In line with this, as part of FfMA's development and refinement, pre-recorded lesson videos were scored by the participating teachers, and the uniformity in these scores was subsequently analysed. Adjustments to FfMA were made based on the congruencies and discrepancies found in these scores. Furthermore, different activity videos were assessed using FfMA by the same evaluators at varying intervals to examine the uniformity of the scores.

To enhance the sensitivity of the instrument, indicators for every component related to both activity design and implementation within FfMA were delineated, with scoring being conducted based on these indicators. In addition, illustrative explanations were appended to each scoring descriptor within the tool to clarify the scoring guidelines. After every refinement phase of FfMA, the tool was tested through teachers' actual implementations, which were then video recorded. Independent from the teachers, researchers also evaluated these videos. The results of their evaluations were juxtaposed, followed by deliberations with the evaluators. Through these consultations, the final rendition of FfMA was established, incorporating requisite adjustments, clarifications, examples, and guidelines to ascertain uniformity in scoring.

The usability of a measurement tool pertains to its simplicity, cost-effectiveness, and efficiency in aspects such as access, preparation, development, implementation, and evaluation. Throughout FfMA's development, a primary concern was its user-friendliness. To bolster FfMA's usability, indicators and illustrative examples corresponding to each component were incorporated. Moreover, continuous refinements were undertaken, emphasizing FfMA's efficacy, practicality, and utility. In line with scoring, feedback was consistently solicited from teachers, particularly highlighting aspects that were either unclear or time-consuming during the evaluation process. Consequently, the tool was tailored to be both straightforward and economical in its execution and evaluation.

Feedback from the participating teachers affirmed the usability of FfMA. In both individual and focus group interviews, teachers expressed that they found FfMA beneficial. They mentioned that the tool was not cumbersome; after a few attempts, they became familiar with its use. Moreover, they noted that evaluations using FfMA were time-efficient and that the tool was memorable. Collectively, this feedback underscores that teachers deemed the tool as practical and valuable.

CHAPTER 4

MATHEMATICAL ACTIVITY DESIGN AND IMPLEMENTATION MODEL: A CONCEPTUAL FRAMEWORK

This chapter outlines the conceptual framework that informs the development of FfMA. Situated within the context of design-based research, this theoretical construct encompasses key elements related to FfMA's evaluation and feedback functions. The framework adopts a holistic view of activity design and implementation, integrating relevant concepts and processes with an emphasis on evaluation and feedback. Therefore, this model is expected to provide a better understanding of the evaluation and feedback nature of FfMA and guide the development of FfMA.

In this chapter, we introduce a model formulated with an inspiration of grounded theory approach (Chun Tie et al., 2019). This model emerged from teachers' insights on the preparatory processes for activity-based teaching, classroom practice observations, and reflective interviews with participants post-implementation. Before delving into the model, we clarify how we have approached the concept of activity in this study and share the foundational definition that informed the model's development.

4.1. Operational Definition of Mathematical Activity

Studies on mathematical activities present a wide range of definitions, explanations, and descriptions. While some researchers, such as Ainley et al. (2006) and Griffin (2009), concentrate on identifying characteristics that contribute to the quality of these activities, others focus on the theoretical foundations that guide their design. This is especially true in specialized contexts like mathematical modelling or specific teaching theories like the anthropological theory of didactics (Van Dooren et al., 2013; Kieran et al., 2015). Yet another group of researchers emphasizes particular facets of activities, such as cognitive demand, context selection, and real-world applicability (Henningsen & Stein, 1997; Smith & Stein, 1998; Clarke & Roche, 2018). Despite the considerable body of research on mathematical activities, there is a lack of operational definitions that could facilitate comparative analyses of research findings, empirical studies, and evidence-based evaluations. This lack points to the pressing need for such definitions, which are crucial for establishing guiding theoretical or conceptual frameworks in the field of mathematical activity research.

In this study, we present an operational definition of mathematical activities, derived from an exhaustive examination of textbooks from various countries, insights from academics and practitioners, and a comprehensive review of relevant literature. The definition is as follows:

> *A mathematical activity encompasses one or a coordinated sequence of scripted tasks that are manageable for students, necessitating their active engagement and enhanced with appropriate materials, with the intention of producing an outcome.*

This definition was formulated based on six core attributes that relevant research shared thus far suggests are essential for effective and enriching mathematical activities. Each of these attributes is inherent in the definition provided above and becomes evident when the definition is dissected. We elaborate on each of these characteristics below.

1. Core of Mathematical Activities: Scripted Tasks

Mathematical activities are not mere assignments or exercises. A mathematical activity could be best described as an experience built on a solid foundation. The term “scripted tasks” implies that there is a predefined structure or ‘script’ that has been thoughtfully planned in advance.

2. The Structure of an Activity: Single or Coordinated Multiple Tasks

At its core, every mathematical activity holds the promise of exploration and discovery. But the pathways to these revelations can vary in complexity and depth. In designing mathematical activities, singular tasks provide clarity and precision, while coordinated sequences offer breadth and integration. Whether it is a single task or a connected series, each is intricately woven to ensure coherence and progression.

3. A Student-Centric Design: Manageable Tasks

Central to any effective mathematical activity is the student’s experience. Each activity is crafted to ensure it is manageable, striking a balance between challenge and achievability. By aligning tasks with students’ capabilities, it is important to ensure that they feel empowered and confident.

4. Fostering Engagement: Active Participation

Mathematical activities should never be passive. They must demand active student engagement, putting learners at the centre of their own development. This emphasizes that students are not mere recipients of knowledge but are actively involved, responsible for their own learning and growth.

5. Augmenting the Experience: The Role of Materials

An effective mathematical activity is often paired with enriching materials. Incorporating appropriate resources adds depth and context, ensuring that tasks are not merely theoretical but rooted in tangible experiences and/or real-world applications.

6. Purpose-Driven Learning: The Outcome

While every activity has its unique characteristics, the concept of achieving an "outcome" remains universal. Each mathematical task is geared toward an end goal or objective, guiding learners toward a specific result or deeper understanding. The outcome, though not always tangible, provides purpose and direction.

The definition outlined here serves a practical purpose, laying out essential concepts for empirically studying mathematical activities. It also underpins the theoretical framework that we call Mathematical Activity Design and Implementation Model, which we detail in the next section. This model has significantly influenced how we assess the quality of activity-based teaching practices and has helped shape the aspects and components of FfMA, offering valuable insights for teachers and researchers Both the definition and the model, along with FfMA, are grounded in empirical research and a detailed review of existing literature, together providing a comprehensive and integrated view of activity-based mathematics instruction.

4.2. Mathematical Activity Design and Implementation Model

The "Mathematical Activity Design and Implementation Model", depicted in Figure 4.2.1, serves as the theoretical framework underpinning the development of FfMA. This model provides guidance on the various dimensions involved in the process of activity design and implementation, as well as a comprehensive evaluation of these dimensions. Furthermore, the model plays a pivotal role in elucidating the structure and inherent attributes of FfMA. This section outlines the model to better understand the theoretical background and its operational principles.

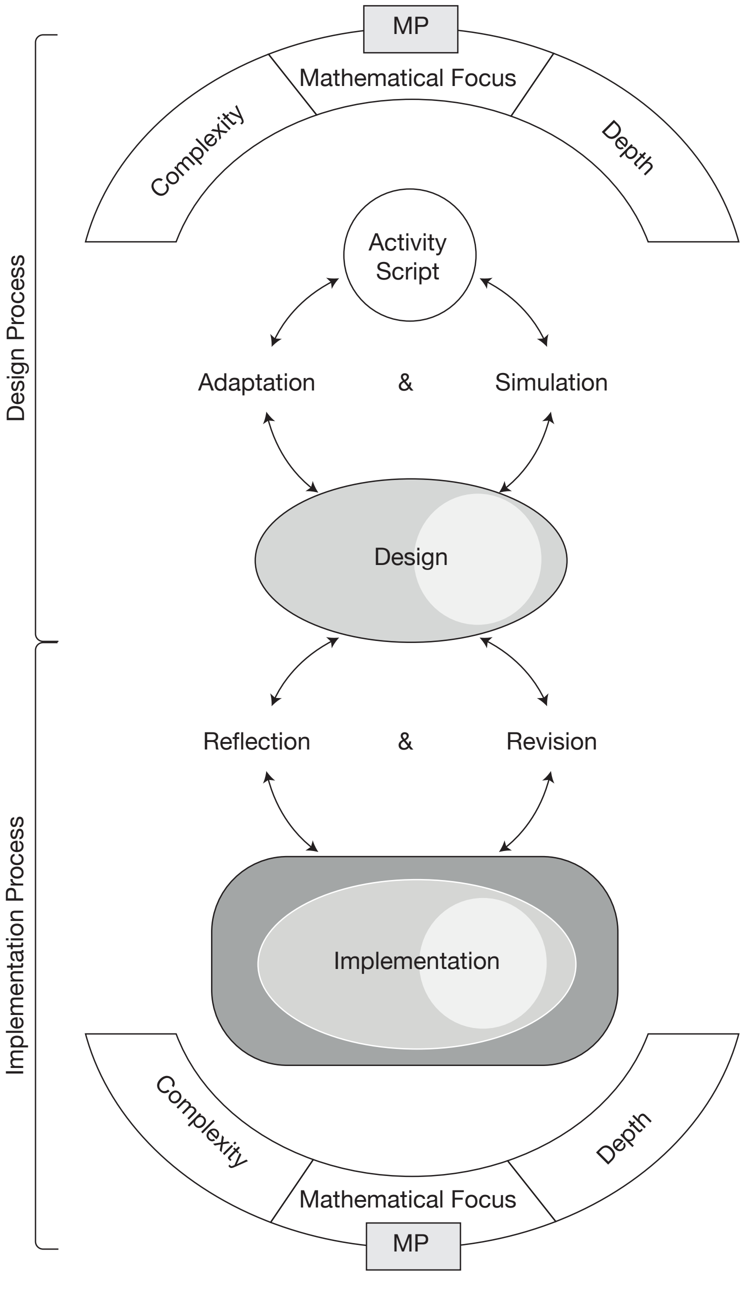


Figure 4.2.1 Mathematical Activity Design and Implementation Model

The model depicted in Figure 4.2.1 breaks down activity-based instruction into two main stages: design and implementation. The design stage involves either crafting a new activity by the practitioner(s) or selecting one from various sources like textbooks, curricula, or social media posts. It also includes preparatory work aimed at adapting and simulating the chosen activity for implementation in an actual classroom setting. The implementation stage is where the designed activity is integrated into the instructional practices. This stage not only involves executing the activity but also includes reflections and revisions aimed at enhancing its quality. In the following sections, the essential concepts and elements of these two stages are briefly described.

Activity Script: An activity is comprised of either a single task or a series of interrelated tasks, all designed to achieve a specific goal. The activity's script outlines the execution scenario, complete with instructions for carrying out the designated task(s). This script must detail the information to be conveyed to the students during implementation and align with the intended outcome. The finalized version of this script, ready for deployment, emerges from the design process. Ideally, the task outlined in the activity script should lead to a particular mathematical development on the part of students.

Design: Activity design refers to the formulation or selection of a task scenario that is prepared for execution. While some resources may offer pre-scripted activities, it is essential for teachers to adapt them to fit their specific classroom context. The design process transforms an activity—whether sourced externally or created by the teacher—into one that is suitable for actual classroom implementation. This transformation is achieved through repeated cycles of adaptation and simulation.

The Interplay of Activity Script and Design through Adaptation and Simulation: The intricate relationship between the activity script and design can be distilled into two processes: adaptation and simulation. Adaptation entails making necessary modifications to an activity script, taking into account specific classroom conditions and student needs. On the other hand, simulation involves the teacher mentally envisioning how the adapted activity would unfold in the classroom setting. This iterative cycle ultimately results in a refined activity script ready for deployment. Illustrated as a recurring loop in Figure 4.2.1, these processes represent a dynamic feedback system. The adaptation and simulation cycle repeats until the teacher is assured of a successful execution. Throughout this iterative cycle, the teacher continuously refines the activity, making informed decisions about further adaptations and revisions to the activity's script.

Implementation: This phase involves executing the carefully designed activity. Here, the task as scripted is carried out, guided by the teacher's pedagogical strategies which were decided upon during the design process. The implementation stage is a dynamic interplay actively influenced by the interactions among the student, teacher, and the mathematical content.

The interplay of Activity Design and Implementation through Reflection and Revision: The interplay between the design and implementation phases can be best described through the concepts of reflection and revision. Reflection involves the teacher's assessment of the classroom experience, either during or after the activity has been implemented. This evaluation identifies the strong and weak points of the implementation process, and informs whether adjustments to the design, including activity selection, are necessary. Any subsequent changes that the teacher decides to make based on this reflective evaluation are termed as revisions.

Mathematical Potential (MP): Mathematical Potential (MP) is a key dimension in the model presented in Figure 4.2.1, and it is evaluated both in the activity script and the implementation phases. MP highlights the importance of domain-specific attributes in activities designed for mathematics instruction. In this framework, an activity's mathematical potential is built upon three main features: depth, complexity, and mathematical focus. Depth refers to the quality of understanding that the activity affords regarding underlying mathematical principles and generalizations. Complexity involves the interrelation of mathematical structures and ideas from various angles, such as through different time frames, academic disciplines, or representations. Mathematical focus refers to the clarity and explicitness of mathematics embedded in an activity. These features will be further explored in the section where the framework for FfMA is introduced. In essence, these three serve to ensure that the activity and its implementation offer a rich mathematical experience.

The Mathematical Activity Design and Implementation Model provides a holistic and practical framework for understanding activity-based instruction. It encompasses both the design and implementation phases, as well as the transitional processes between them, captured through concepts of adaptation-simulation and reflection-revision. The model's dynamic and dialectical nature elucidates the interrelated aspects of scripting, designing, and implementing activities. This is instrumental for highlighting the comprehensive nature of activity-based instruction, pinpointing key areas for evaluation, and identifying stages where opportunities for development and improvement are implicit.

CHAPTER 5

MATHEMATICAL ACTIVITY EVALUATION AND FEEDBACK TOOL

In this section of the book, we share the details of the dimensions, components, and specific content of each component within FfMA, along with associated indicators. Moreover, each component has been assessed and ranked based on established criteria. Before we unpack these details, we provide a concise overview of FfMA and shed light on its inherent structural characteristics.

5.1. F*f*MA and Its Structural Characteristics

FfMA serves as an evaluative tool designed to independently assess the quality of both the activity script and its subsequent implementation. The primary objective is to utilize the script of the mathematical activity as a means of providing targeted feedback on the strengths and areas for development that emerge during the activity's execution. Constructed as an analytical rubric, FfMA is employed to evaluate individual components that constitute the overall performance, thereby allowing for a nuanced analysis of each component.

One of the key advantages of using analytical rubrics lies in their capacity to offer specific feedback for each performance component, in accordance with predefined scoring criteria. This granularity facilitates a more comprehensive understanding of both strengths and weaknesses. Moreover, by breaking down holistic evaluations into smaller, more manageable components, analytical rubrics enhance the clarity and objectivity of the scoring process (Sadler, 2009), thereby bolstering confidence in the evaluation outcomes.

In the assessment process, each component is evaluated separately using the analytical rubric, and their respective scores are subsequently aggregated to derive a cumulative score. Consequently, FfMA's analytical rubric structure enables a more detailed and customized assessment. It also allows for the computation of a composite score related to both the activity's design and implementation by summing the individual component scores obtained during the evaluation process.

For FfMA to serve effectively as an evaluation tool, it relies on tangible indicators and observable criteria. This ensures that FfMA not only provides reliable scoring but also yields valid results.

FfMA assesses two main facets: the activity script and the implementation process. The activity script, either extracted from various sources or crafted by teachers, possesses tangible and observable qualities. In contrast, the implementation emerges from the real-time interaction between the student, teacher, and mathematical content in an actual classroom setting, exhibiting its own set of observable characteristics. The dimensions and components of FfMA are illustrated in Figure 5.1.1.

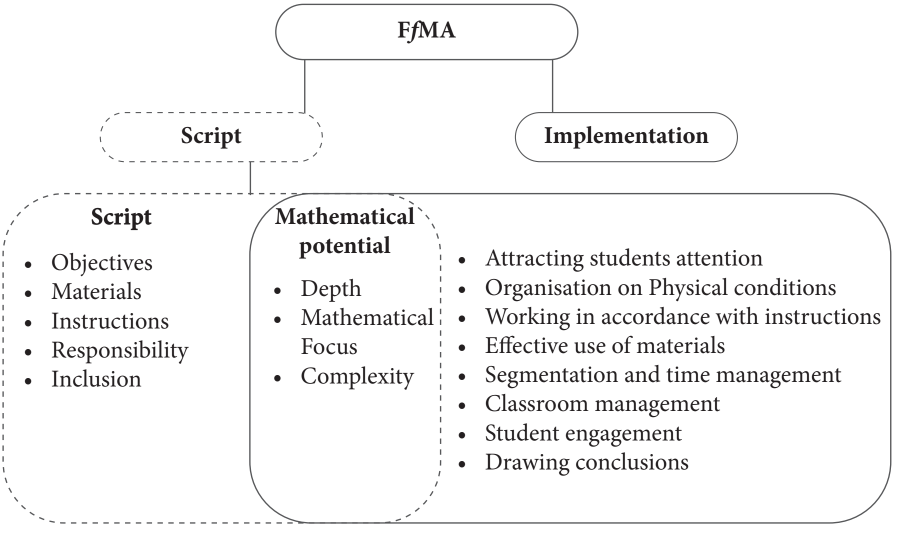


Figure 5.1.1. Dimensions and components of FfMA

As illustrated in Figure 5.1.1, the activity script is comprised of 8 components, while the implementation includes 11 components. Both dimensions, the activity script and the implementation, are evaluated with respect to their mathematical potential. This potential encompasses components related to the activity script as well as the mathematical characterization of the implementation. Three key components assess the quality of the intended mathematics within the scope of the activity: Mathematical focus, depth, and complexity. Both the activity script and the implementation dimensions should be viewed in conjunction with these three.

The components in the FfMA are rated over 4 score types (0: Vey Low; 1: Low; 2: Moderate; 3: High). This scoring system in FfMA provides a clear and quantifiable measure for evaluating the activity script and the implementation process. By utilizing a range from "Very Low" to "High," it allows for a nuanced understanding of the quality of both the design and execution of a given mathematical activity. With a potential score range of 0-24 for the activity script and 0-33 for the implementation, this scoring system ensures that each component is given equal weight in the assessment, making it comprehensive and balanced. The granularity of this approach can also offer specific feedback, highlighting areas of strength and opportunities for improvement.

In the FfMA, each component within the activity script and the implementation phase is independently evaluated, allowing for variance in scores based on specific criteria for each component. The objective is to attain the highest possible score across all components, both in the activity script and in its subsequent implementation. It is crucial to clarify that achieving high scores does not imply that the activity is solely suitable for students with advanced cognitive abilities. Rather, the emphasis is on designing the activity script in alignment with FfMA guidelines and adapting it skilfully for effective implementation, regardless of the cognitive capabilities of the target audience. Consequently, the criteria for high scores in the FfMA are devised to underscore the need for quality in both the design and implementation of the activity. These criteria advocate for a flexible approach that accommodates diverse learning needs, thereby ensuring that the activity maintains its efficacy, even when adapted for different audiences.

5.2. Dimensions and Components of F*f*MA

The FfMA is structured as a rubric encompassing three primary dimensions: the activity script, activity implementation, and mathematical potential. Each dimension is further delineated into specific components. For each component, there is a concise description, relevant indicators, criteria for scoring, detailed explanations, and illustrative examples. This detailed presentation serves to clarify the scoring process and considerations for each component. For practitioners seeking a more condensed reference, an abridged version of the FfMA is available in the appendices of this book (refer to Appendices 1 through 5). This streamlined format is designed for ease of use and quick reference.

5.2.1. Components of Activity Script

Activities are typically presented as textual documents featuring scripted tasks that are driven by specific instructions. These documents not only detail the activity itself but also provide clear directions for its execution. We have identified five primary components in evaluating the quality of activity scripts: objectives, instructions, materials, responsibility, and inclusion. Below are brief descriptions for each of these components.

1. **Objectives**: This component specifies what the activity aims to achieve, namely which learning outcomes it targets. Clear and specific objectives guide both teachers and students, establishing what they should recognize as a successful outcome at the end of the activity.
2. **Materials**: This component outlines all tools and resources required to conduct the activity. The selection of the right materials plays a critical role in the seamless and successful completion of the activity.
3. **Instructions**: Instructions guide students on how to carry out the activity. An effective activity script includes clear, understandable, and step-by-step instructions, leaving no ambiguities for students.
4. **Responsibility**: This component focuses on the allocation of tasks and duties, ensuring that every participant knows their specific role within the activity and what is expected of them.
5. **Inclusion**: This addresses whether the activity is inclusive and encompassing of all students. The inclusion component ensures that every student takes an active role and is fully engaged in the learning process.

In subsequent sections, we will explore each of these components in greater detail. We will present rating criteria and indicators to guide the evaluation process. By gaining a thorough understanding of these components, teachers can more effectively select or design mathematical activities.

Objectives

Each activity is centred on a specific, attainable objective. This objective refers to the mathematical understanding or growth anticipated from the student(s) upon completion of the activity. Within the context of the activity, the objective represents the mathematical idea or message being conveyed to the students. For this message to be effective, the intent of the activity must be clear to its participants. Evaluating the clarity of this objective hinges on two main indicators. The first is **clarity** which emphasizes the precision in articulating the desired outcome of the activity. The second is **comprehensibility that** measures the simplicity and understandability of statements related to the activity's targeted outcome. Together, these indicators are instrumental in gauging the strength of the objective(s) an activity sets out to achieve.

Component	**Objectives:** Refer to the mathematical understanding or growth anticipated from the student(s) upon completion of the activity			
Indicators	• **Clarity**: The activity's objectives are presented without any ambiguity or vagueness. • **Comprehensibility**: The ease with which participants grasp and interpret statements about the activity's targeted output.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The activity's objective is presented with ambiguity, making it difficult for participants to grasp.	The activity's objectives are multiple and uncorrelated, making it challenging for participants to discern the primary intended outcome.	Even if the objective seems unclear at the start, following the instructions or completing the steps clarifies it.	The objective is clearly presented without ambiguity and easy to grasp.
Explanations and examples	• Upon completing the activity, the students' expected achievement remains unclear. • The activity's instructions lack a cohesive or unified objective.	• The main objective of the activity aligns with multiple curricular outcomes, causing uncertainties about the prioritized outcome. • The primary goal of the activity invites diverse interpretations.	• Understanding the activity's targeted objective initially poses challenges, but following the instructions brings clarity to it.	• Preliminary information is provided about what the activity intends to achieve.
Key considerations for proper scoring	• Focus on the evaluation of the intended objective if it is directly stated within the activity text. • If the activity's intended objective is subtly placed within the instructions, interpret the objective using those directions. • It is crucial to ascertain how the activity will end when instructions are followed for objective assessment. • Once clarity on the activity's intended objective is achieved, review the instructions with this perspective and score based on the established criteria.			

Materials

This term encompasses both digital tools and tangible objects utilized in learning and teaching. They can include various media forms such as printed materials, visual and auditory records, as well as tools aiding in instruction. At their core, these materials are designed with a pedagogical purpose in mind. Two primary indicators determine the effectiveness of these materials: essentiality and practicality. The first indicator, **functionality,** assesses the material's significance for successful implementation. It delves into the essentiality of the material in achieving the intended objective, its indispensability, and what might be lost in its absence. **Practicality** is the second indicator which relates to the material's ease of use without any complications. It ensures that the material is free from potential issues when used within an activity, ensuring safety, avoiding time wastage, and preventing confusion.

Component	**Materials:** Encompass any kind of digital and tangible tools/objects aiding in instruction.			
Indicators	• **Functionality**: Gauges the material's crucial role in achieving objectives and the impact of its absence. • **Practicality**: Assesses the material's user-friendliness and efficiency.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The material is neither functional nor practical.	The material is practical but not functional.	The material is functional but not practical.	The material is both functional and practical.
Explanations and examples	• The material does not significantly contribute to the intended objective. Its application introduces challenges or complications in the activity.	• The material does not play a central role in achieving the intended objective. Alternate materials could better serve the purpose. • Its use might introduce misconceptions.	• The material, while beneficial for the objective, has practical drawbacks. • Despite its alignment with the objective, the material's use can result in wasted time, safety concerns, or classroom management challenges.	• The material is both directly relevant to the intended objective and easy to use. • It serves its purpose without posing unnecessary challenges or risks.

	• For example, utilizing a scientific calculator in an activity intended to teach basic arithmetic operations to students at primary level.	• - For example, using halves of two differently sized apples in an activity comparing fractions might lead to confusion.	• For example, asking students to cut numerous squares from cardboard using a utility knife, which can be time-consuming and pose safety risks.	• For example, using manipulative base-ten blocks to teach place value, providing a hands-on understanding for students without any complications or hazards.
Key considerations for proper scoring	• When evaluating an activity, carefully examine its entire text to understand which materials are required. Some activities may list materials explicitly, while others may imply material needs within the instructions. • Take into account how the materials will be used in the activity and who will be responsible for their usage. This can impact both functionality and practicality assessments. • Evaluate the materials first for their functionality in achieving the activity's objectives and then assess their practicality. • Consider the logistical aspects of the materials: Will they be pre-prepared, or will they need to be assembled during the activity? Who will supply them? These factors should be assessed for their impact on practicality.			

Instructions

The instructions within an activity are designed to outline the task, communicate expectations to students, and establish a framework that guides them toward the desired performance level or learning outcome. When followed, these instructions facilitate the student's achievement of the targeted outcome or aid in comprehending the intended objective. Thus, instructions serve as a set of directions to think and act in certain ways.

The quality of instructions is evaluated based on three primary indicators: easy-to-follow, relevance, and alignment with the objective. For instructions to be deemed **easy-to-follow**, they need to be articulated in clear, simple, and comprehensible language, free from redundant information or processes that are extraneous to the activity's purpose. **Relevance** requires the instructions to directly pertain to and serve the main goal of the activity. **Alignment with the objective** underscores that when followed, the instructions should guide learners to successfully achieve the target learning outcome.

Component	**Instructions:** Delineate the task, convey expectations, and guide students towards the intended learning outcome.			
Indicators	• **Easy-to-follow**: Instructions articulated in a clear and simple manner, devoid of extraneous or redundant information. • **Relevance**: Instructions that directly relate to and serve the primary goal of the activity. • **Alignment with the objective**: Guidelines that, when followed, steer learners toward achieving the target learning outcome.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The instructions are not easy-to-follow, making it impossible to determine whether the activity's objective has been achieved or not.	The instructions are easy-to-follow, but there are issues or ambiguities that hinder the achievement of the activity's objective.	The instructions are easy-to-follow and offer the potential to achieve the activity's objective, but they contain some superfluous or unrelated steps.	The instructions are easy-to-follow, relevant, and fully aligned with achieving the objective.
Explanations and examples	• The objective remains unattainable due to unclear and ambiguous instructions. • Ambiguities in the instructions prevent their successful execution. • Some instructions include tasks that are either unclear or infeasible. • For example: The vagueness of an instruction like	• Transitional gaps or uncertainties within the instructions hinder the attainment of the objective. • For example, imagine an activity where students are asked to solve a two-step algebraic equation. The first instruction asks students to isolate the term with the variable on	• The instructions contain directives that are not immediately aligned with the main objective. For example, in an activity intended for ordering rational numbers, students are instructed to rank integers as well.	• The instructions are interconnected and guide towards accomplishing the intended objective.

	• "Determine what percentage of a building under construction in your neighbourhood has been completed" makes it challenging to ascertain the student's percentage calculation accurately.	• one side, but the second instruction jumps to asking students to substitute a value for the variable.		
Key considerations for proper scoring	• While instructions in activity texts are typically itemized, sometimes directives or descriptions might be dispersed throughout the text. It is essential to discern these instructions from the provided descriptions. • Instructions often align with different phases of the activity. For relevance, it is crucial to ensure that these instructions facilitate connections both within and between these stages. • When encountering unrelated instructions, evaluate their impact on achieving the objective. Based on this assessment, determine if the score should be low (1 point) or moderate (2 points).			

Responsibility

In the activity script, 'responsibility' delineates not just the duties but also the working styles that students are expected to adopt throughout the course of the activity. This includes detailed guidance on what is required from students in terms of works, the approaches they should employ, and the specific roles they need to assume to optimize the opportunity for achieving the targeted learning outcome. These specified roles are designed to align closely with the particular objectives and working procedures of the activity. Further to this, the instructions should clearly define the roles and working approaches students are expected to embrace.

In assessing the "responsibility" component of an activity script, two aspects become decisive: role definition and student engagement. The **role definition** stresses the significance of having well-defined duties and expectations for students. For an activity to be effective, it is paramount that student roles are not only clearly outlined but also presented in an understandable manner. Ambiguity can hinder a student's understanding and performance; hence it is essential to avoid vague descriptions regarding student responsibilities within the activity's framework. The **student engagement** pertains to the structure and presentation of these roles in a manner that fosters active participation. By ensuring that students are motivated and encouraged to actively contribute, we enhance their investment in the learning process. Consequently, when roles are structured to support active participation, students are more likely to put forth the necessary effort to achieve the desired learning outcome.

Component	**Responsibility:** Describes the duties and working styles that students are expected to adopt throughout the course of the activity.			
Indicators	• **Role Definition**: Refers to the clear and understandable outlining of duties and expectations for students. • **Student Engagement**: Emphasizes the importance of structuring roles to promote active participation.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Responsibilities assigned to students are not delineated or specified.	Student responsibilities are outlined, but they are presented in an ambiguous or unclear manner, leading to potential misunderstandings.	Student responsibilities are well-defined and clear, but the structure or presentation limits the active engagement and involvement of the students in the process.	Student responsibilities are clearly and precisely defined, encouraging active participation and engagement throughout the process.
Explanations and examples	• The role of students within the activity remains unspecified. • The expected working styles or approaches for students are not articulated.	• While there's mention that students should collaborate, the method or manner of collaboration remains undefined. • The nature of the students' collective work or their shared responsibilities	• The duties, responsibilities, and working styles expected of students are articulated. Nonetheless, the way these roles and responsibilities are allocated may not ensure active involvement	• The configuration of students' working styles and roles is deliberately designed to enhance their engagement. • Clear guidelines are provided regarding

	• It remains ambiguous as to who is tasked with executing the instructions. • Example: An activity instructs, "Solve the given problem." However, there's no clear indication of which group or individual is expected to take on the task or how they should approach the solution.	within the activity is left to interpretation, leading to potential confusion. • Example: • An activity instructs, "Work together to analyze the given data." Yet, there's no clarity on how the students should divide the tasks, whether they should discuss or document their findings, or how their collaborative effort should be organized.	from each student, potentially leaving some students on the sidelines. • Example: • An activity instructs: "In your group, one student should collect data, and the others should analyze it." This does not give each student an equal chance to partake in both data collection and analysis, thereby limiting active participation in all facets of the task.	the specific roles students will assume during the activity. These roles ensure every student has the intellectual involvement and/or pathway to achieve the desired outcome. • Example: • An activity directs: "In your team, rotate roles where one student presents the findings, another analyses the data, and another records the process. Each member should experience all roles by the end of the activity." This approach guarantees each student actively engages in every facet of the task.

Key considerations for proper scoring	• While activities generally detail the steps to be taken, a crucial factor is the clarity regarding responsibility assignment. It is vital to ascertain who is responsible for what task, how the task is to be executed, and in which manner. Statements crafted using passive verbs frequently introduce ambiguities. For instance, in the phrase, "the width and height of the class are measured with the help of a tape measure (1 point)", it remains ambiguous as to who will perform this task and how it will be done. • Active student engagement is pivotal to enhancing the quality of activities. Thus, defining responsibilities in a manner that embeds students into the process is essential. To achieve moderate or high scores, the emphasis should be on involving students actively in the task.

Inclusion

Inclusion ensures that an activity caters to a diverse range of students at various developmental stages. Such activities are designed to engage all students, taking into account their diverse backgrounds, be it gender, cultural, academic, ethnic origin, or socio-economic status. Structured tasks with varying levels of difficulty are crucial to achieve this inclusivity. By addressing both the needs of slower learners and those with quicker comprehension, the activity ensures every student can engage based on their individual capabilities, preventing any one group from dominating. For an activity to be truly inclusive, it should have a low entry or starting threshold, enabling broader student accessibility.

Component	**Inclusion:** Ensures that an activity caters to a diverse range of students at various developmental stages.			
Indicators	• **Diverse Accessibility**: The attribute of an activity that ensures inclusivity, accommodating students from varied backgrounds such as socioeconomic, cultural, academic, gender, and ethnic origins.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The activity remains unable to accommodate the majority of the students.	The activity accommodates students with specific characteristics only.	While the activity accommodates most students, some from varied backgrounds remain uninvolved.	The activity ensures full diverse accessibility, accommodating all students.

Explanations and examples	• Contexts unfamiliar to the majority are incorporated. For example: Using a lesser-known card game without providing clear rules. • The activity's starting point is considerably beyond the developmental level of most students.	• The activity targets specific interests. **For instance:** Requiring football knowledge for progression. • It caters to a particular academic achievement level, where either high or low achievers predominantly benefit. • The activity is designed around the contributions of a select group. **For example:** A majority of the class merely observes as a select few complete their designated tasks.	• Measures are in place to avoid direct discrimination; however, unintended disadvantages can still arise. **For instance:** An activity that uses weight information may unintentionally deter overweight students from participating.	• The activity ensures equal access and participation for all students regardless of cognitive level, gender, cultural background, and socio-economic status. • Design features of the activity accommodate a diverse range of students. **For instance:** The inclusion of open-ended questions and multiple entry points.
Key considerations for proper scoring	• Inclusivity emphasizes ensuring every student has equal opportunities to access and engage with the activity. When scoring, the focus should be on whether elements of the activity inherently exclude students from the outset, rather than if a student chooses to leverage the given opportunity during execution. • Inclusiveness and student engagement are distinct concepts. The former is evaluated solely from the activity’s text, whereas the latter pertains to its implementation. • Inclusivity directly aligns with principles of social justice and inclusion. Activities should be critically examined to identify if they inadvertently disadvantage any student or group. Activities that cater only to high achievers, for instance, should receive a low score for inclusivity. • The potential number of students excluded from the activity plays a significant role in determining the score.			

5.2.2. Components of Activity Implementation

To evaluate the quality of activity implementation, we have identified eight foundational components. Each of these plays a significant role in ensuring the effectiveness and efficiency of the activity, as described below:

1. **Attracting Student Attention:** It is essential to keep students engaged and attentive. Activities should be designed to capture and maintain their interest from start to finish.
2. **Organisation of Physical Conditions:** The setup of the classroom or learning environment can greatly impact the activity. Proper space, seating arrangements, and easy access to resources are crucial for an optimal learning experience.
3. **Working in Accordance with Instructions:** Clear and understandable instructions are key. Students should be able to follow the activity's guidelines without any confusion.
4. **Effective Use of Materials:** The choice and use of materials, whether digital, tangible, or hands-on tools, should align with the activity's goals and be used effectively throughout the process.
5. **Segmentation and Time Management:** Breaking the activity into manageable parts ensures smooth progression. It's vital that each segment is given adequate time, and the overall pacing of the activity aligns with the allocated timeframe.
6. **Classroom Management:** It's important to monitor and ensure that students remain on track. Prompt interventions can address any misunderstandings or disruptions, allowing for smooth progress.
7. **Student Engagement:** Activities should be implemented in a way that promotes and sustains active participation from all students throughout the session.
8. **Drawing Conclusions:** Encouraging students to draw conclusions or infer meanings from the activity is crucial. The goal is to achieve mathematically relevant insights and understanding.

In the following sections, we will delve deeper into each of these components, providing rating criteria and indicators that guide the evaluation process. By understanding and appreciating these aspects, teachers can ensure a comprehensive and effective implementation of mathematical activities.

Attracting student attention

For students to be fully immersed in the activity implementation, their attention must be diligently directed towards the activity. The objective is to create an environment where students are not only intrinsically motivated to engage but also eager to contribute both mentally and physically to the task. To achieve this, teachers should aim to stimulate genuine interest, ensuring students remain active and involved. Emphasizing the importance and relevance of the activity further aids in this, making students recognize its value and feel an intrinsic need to participate. Additionally, clearly articulating the activity's purpose ensures students perceive its significance in their learning process, fostering deeper engagement and commitment. By seamlessly integrating these elements into their practices, teachers can create an environment where students are both attentive and motivated, enhancing the overall effectiveness of the activity implementation.

<table>
<tr><th>Component</th><th colspan="4">Attracting student attention: Entails guiding their engagement, sparking genuine interest, and highlighting the activity's relevance to enhance motivation and commitment.</th></tr>
<tr><td>Indicators</td><td colspan="4">• Interest Stimulation: Strategies used by teachers to initiate and maintain student curiosity throughout the activity.
• Significance Highlight: The teacher's emphasis on the importance and relevance of the activity.
• Felt Necessity: Strategies aimed at making students feel the need to engage in the activity.</td></tr>
<tr><td>Score</td><td>Very Low (0)</td><td>Low (1)</td><td>Moderate (2)</td><td>High (3)</td></tr>
<tr><td>Criteria</td><td>No visible attempts to stimulate student interest, emphasize the activity's relevance, or establish its necessity.</td><td>Relies mainly on pro-forma (standard/ formal) expressions for engagement. Limited emphasis on the activity's importance and necessity.</td><td>Efforts go beyond mere pro-forma expressions to engage students, but the impact across the class remains limited.</td><td>Effectively stimulates genuine interest, emphasizes importance, and ensures students perceive the activity's necessity.</td></tr>
</table>

Explanations and examples	• There's no discernible attempt to stimulate interest, convey the importance, or create a perceived need for the activity, preventing students from feeling involved.	• Reliance on pro forma (standard/formal) statements like "we will cover an important topic" or "you will love this activity" as the primary means to pique students' interest.	• The teacher tries to captivate students, but there's a noticeable lack of widespread enthusiasm or involvement. For instance, while the importance of a topic is articulated, it evokes minimal reactions or queries from the students.	• It's evident that the teacher's strategies to spark interest, establish the activity's necessity, and stress its importance are successful, as demonstrated by the students' evident enthusiasm and involvement.
Key considerations for proper scoring	• While scoring, note that the teacher's strategies to engage students may not occur at the beginning of the activity. Any efforts to stimulate interest or emphasize importance at later stages should also count towards the evaluation. • The level of students' enthusiasm, their questions, or active participation throughout the activity should serve as valuable factors in the scoring process. These reactions can offer insight into the effectiveness of the teacher's engagement strategies. • If the activity or associated tasks captivate students' attention on their own, this should also be credited as part of the teacher's success in focusing student attention. Observable student interest should positively impact the final score.			

The organisation of physical conditions

The organisation of physical conditions is pivotal in ensuring the classroom environment is conducive for the successful implementation of activities. This involves strategically arranging furniture such as desks, chairs, and other relevant items. Depending on the nature of the activity, the seating might be U-shaped, double session, "round" table for group tasks, or may require a free space at the front for an activity involving the entire class. Such an "Activity-Focused Arrangement" ensures students can work productively, fostering both collaboration and individual engagement. Furthermore, resources within the classroom, like computers and wall panels, should be accessible and ready for use. If an activity demands these resources, appropriate adjustments should be made to facilitate easy access, promoting a seamless flow during the session. Another significant aspect to consider is how these physical conditions or arrangements support the diverse working styles of students. An optimal learning environment recognizes these differences and provides arrangements that cater to varying student needs, thereby enhancing their overall learning experience.

Component	**The organisation of physical conditions:** Refers to the strategic arrangement of classroom furniture and resources to support optimal student engagement and activity implementation.			
Indicators	• **Activity-Focused Arrangement**: The classroom setup, including furniture and seating, is tailored to the specific activity's needs. • **Supportive Layout**: The physical layout supports the specific working styles prescribed by the activity.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The classroom setup noticeably hinders the activity's objectives, making the completion of the activity impossible.	While the activity proceeds, the environment's physical conditions cause significant challenges or interruptions during the execution.	The classroom's physical conditions may not be fully tailored for the specific activity, but they do not obstruct its execution.	The classroom's physical setup perfectly aligns with the activity's requirements, ensuring a smooth execution.

Explanations and examples	• There is a clear misalignment between the activity's requirements and the provided physical conditions. • The physical environment is so unsuitable that the activity has to be terminated. • Example: The activity required students to form groups in the empty space in front of the board, but it was abandoned because the space proved insufficient.	• Teachers and students need to make immediate adjustments, potentially disrupting the activity's smooth flow. • The physical conditions of the classroom cause significant issues that disrupt the activity, requiring immediate adjustments that impact its smooth flow. • Example: Students initially seated in pairs have to rearrange themselves into groups of four, causing turmoil and distractions.	• Minor adjustments might be made during the activity to accommodate, but they do not significantly affect the flow or outcome. • The classroom is arranged to accommodate the activity, but minor limitations exist that could not be addressed by the teacher. • Example: Desks are too small for the required materials, or the teacher is unable to move freely due to limited space between the desks. Despite these issues, the activity proceeds without significant interruption.	• Both the activity-focused arrangement and the supportive layout are evident, ensuring smooth execution without any physical barriers or challenges. • The physical conditions were well-suited for the planned activity, allowing it to proceed without any hindrances.

Key considerations for proper scoring	• Activities are not restricted to just the classroom. Evaluations should account for alternate venues like laboratories, school gardens, gyms, etc. The scoring is based on the suitability of the physical conditions of the chosen environment for the specific activity. • While various issues can interrupt an activity (e.g., misunderstood instructions, incorrect material usage), it is crucial to determine if disruptions stem directly from the environment's physical conditions. Only issues caused by the environment should impact this component's score. • Evaluators must monitor whether the execution of tasks and responsibilities aligns with the environmental conditions. Assess if students can effectively undertake the given tasks within the available physical setup throughout the activity. • Even if the unfavourable physical conditions cause problems due to reasons other than teacher-student, scoring will be adversely affected. For instance, if a class is too small for a group activity, the selection of that activity for that particular space is flawed, and it will reflect in the scoring. Activities should be selected with an acute awareness of the environment's constraints in mind.

Working in accordance with instructions

This component emphasizes the vital role of clear communication of instructions in facilitating effective student participation. Under this component, the assessment focuses on how students align with the guidelines outlined in the activity script as well. For optimal student engagement, it is crucial that instructions are delivered with clarity. This entails leveraging a straightforward and comprehensible language while presenting the guidelines. Techniques such as modeling, exemplification, and demonstration serve as powerful tools to enhance understanding, ensuring that students grasp the expectations placed on them. Additionally, a pivotal facet of this component is student adherence to instructions. This evaluates the extent to which students' actions during the activity mirror the given instructions. Not only is it essential for students to stay on course, but it is also the teacher's responsibility to guide and ensure that students' actions align with the given directives, reflecting interdependent relationship between instructional clarity and adherence.

Component	**Working in accordance with instructions:** Refers to students' consistent alignment with and adherence to provided guidelines during an activity.			
Indicators	• **Clear Communication of Instructions**: Instructions are delivered with clarity and precision to ensure students grasp them fully. • **Student Adherence to Instructions**: Students actively engage in the activity as per the given guidelines.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The instructions were either not communicated or were misunderstood, leading to unrelated tasks being performed or the activity being incomplete.	Instructions were provided, yet a significant portion of the class struggled to comprehend and/or follow them, indicating gaps in the communication or understanding	The teacher clearly communicated the instructions. However, a few students exhibited challenges in aligning their actions with the given guidelines	The teacher explicitly conveyed the instructions, and all students engaged in the activity according to the provided guidelines.

Explanations and examples	• The instructions are presented to the students without any further explanation or verification of understanding. • Example: Students divert	• While instructions are shared and clarified, the teacher does not ensure comprehension before the students begin the activity.	• The instructions are provided and explained. Students indicate they have understood, but some slight deviations from the instructions are observed.	• Instructions are delivered, clarified, and feedback confirmed students' understanding. Any unclear areas are addressed promptly.
	from the task at hand and engage in works that were not directed by the instructions.	• Example: Students frequently expressed uncertainty or asked questions about their tasks, indicating confusion about the instructions throughout the activity.	• Example: While the majority followed the instructions, some students occasionally engaged in works that were not explicitly mentioned or showed minor misunderstandings in their implementation.	• Example: Students consistently followed the given instructions, demosntrating a clear grasp of their tasks throughout the activity.
Key considerations for proper scoring	• Challenges resulting from the cognitive demands of the activity should not factor into the assessment for this component. • Although instructions are typically presented at the outset of the activity implementation, there are occasions when a teacher might opt to share them progressively or at later stages. Not presenting instructions at the outset does not necessarily warrant a low score. • Evaluations should consider the entire duration of the activity. It's important to factor in situations where students might display a lack of understanding only after the activity concludes. • Students may face difficulties even if the instructions are clear. Clarifications sought by students or minor struggles to understand should not negatively impact the score. • Persistent misunderstanding of instructions, leading to ongoing issues throughout the activity, are indicative of potential low scoring. • When scoring, the proportion of students adhering to the given instructions should influence the final score.			

Effective use of materials

This component is critical for ensuring that all materials utilized in the activity implementation are effective tools for achieving the intended mathematical learning outcomes. The materials should serve the dual purpose of facilitating task completion while also enhancing comprehension. Three key indicators come to the fore while evaluating the effectiveness of this component: alignment with objectives, efficiency in task execution and smoot integration. Regarding the first indicator, materials should be directly aligned with the mathematical objective, ensuring they enhance, rather than detract from, the learning experience. They should not become the main focus but rather serve as an aid to reach the desired outcome. Considering the second indicator, the clarity and functional design of materials are crucial for task execution. They should be straightforward in their purpose and offer an intuitive pathway for students to achieve the mathematical objectives without causing confusion or ambiguity. The focus here is on the efficient and clear use of materials that guide students toward the expected mathematical results. Concerning the final indicator, the incorporation and distribution of materials within the activity should not disrupt the classroom order or divert students' attention from the primary learning objectives. Furthermore, materials should be readily available in the learning environment, ensuring a seamless transition during various stages of the activity.

Component	**Effective Use of Materials:** Ensures that materials used in activities are effective tools for achieving mathematical outcomes, simultaneously facilitating task execution and enhancing understanding.			
Indicators	• **Material alignment with objectives**: Materials directly serve the mathematical outcome without distracting. • **Efficiency of Materials in Task Execution**: Materials provide a clear, direct route without causing chaos or confusion. • **Smooth Material Integration**: Materials are introduced seamlessly, without interrupting the implementation or the flow.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The materials do not align with the mathematical objective and cause significant disruptions in the implementation.	While the materials align with the mathematical objective, they cause confusion and problems severe enough to hinder the implementation.	The materials align with the mathematical objective and cause only minor issues that do not significantly disrupt the implementation.	Materials perfectly align with the mathematical objective, and no material-related problems are observed during the implementation.

Explanations and examples	• Students are insufficient in preparing or using the material and could not participate in the activity. • Example: In the activity of graphically representing the relationship between the	• There was confusion or too much time wasted during the preparation/ distribution of the material, hindering effectiveness. • Materials lead to pursuits that go beyond the purpose of the activity.	• Material distribution and introduction are completed in a reasonable time and without any confusion. However, some students, albeit a small number, could not access the materials.	• Material distribution and introduction are completed in a reasonable time and without any confusion. • The materials to be used in the activity are sufficiently available for each group/ student.
	flight time of a paper airplane and the length of the tail, the students were unable to make airplanes. • The material is generally used outside of the activity purpose. • Example: Students use the material as a game tool in an activity where angle comparisons are expected using tangram.	• Example: The materials required for the activity are not available to all students. • Students were distracted because the materials were distributed before the instructions were explained. • Example: Students were dealing with the material instead of listening to instructions.	• Example: Materials are mostly used for the purpose of the activity. However, some students used it independently of the instructions. • Additional example: In group work, some members of the group worked with the material, while others remained uninterested.	• The material is used in line with the purpose of the activity and did not cause any time loss or problems in classroom management.

Key considerations for proper scoring	• If an activity requires students to prepare the materials in class, like cutting out square cardboard, the relevance of this preparation to the main objective of the activity should be assessed. Activities where the preparatory work doesn't directly support the mathematical learning outcome shouldn't achieve a moderate or high score. • There might be situations demanding adaptability, such as substituting colors when the necessary ones aren't available for a coloring task. The scoring should reflect whether strict adherence to instructions prevents students from progressing, or if the teacher provides an effective alternative solution. • Not all materials are tangible. Virtual tools like math software, images, videos, or informational texts also qualify as materials and should be assessed accordingly. • There might be instances where materials yield unexpected outcomes, such as measuring an angle of a randomly drawn triangle and getting a fractional result with a protractor. During evaluation, it's crucial to see if these unexpected situations create issues in the activity's execution.

Segmentation and time management

This component is particularly crucial when discussing activity implementation in order to express the necessity of well-structured progression and optimal time utilization. Successful activity implementation necessitates a seamless integration of various stages, including individual tasks, various group activities, and class discussions. Such integration requires not only smooth transitions between different phases or stages; but also, an appropriate amount of time is allocated to each segment. This allows for a thorough exploration of the content so that students grasp the main ideas or receive the messages robustly.

The effectiveness of this component hinges on two key areas: Adequate time allocation and minimization of time loss. The former refers to the essentiality of granting each activity stage its deserved time. Such an approach ensures that students engage deeply with the content so that they could find an opportunity to achieve a profound understanding. The latter focuses on preventing wastage of precious time on unrelated or non-contributory tasks. It emphasizes the imperative of maintaining the activity's focus and efficiently navigating through potential pitfalls or distractions.

Component	**Segmentation and time management:** Refers to the structured organization and optimal use of time during various stages of an activity.			
Indicators	• **Adequate Time Allocation**: This assesses whether time assigned for each segment of the implementation is both sufficient and effectively optimized to facilitate understanding and productivity. • **Minimization of Time Loss**: This assesses whether time is wasted on irrelevant tasks or inefficient transitions that do not align with the intended learning outcome.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Time spent on the activity is predominantly occupied with unrelated tasks, resulting in the activity being uncompleted.	There are considerable instances where time is wasted on works that don't support the progress in the mathematical outcome.	Some segments of the activity are either over-extended or too rushed, which may hinder the students' grasp of the mathematical outcome.	The time set for each stage and the entire activity effectively supports students' progress in the mathematical outcome, avoiding time-wasting tasks or delays.

Explanations and examples	• Students were engaged in unrelated activities during the designated study time. • The activity remained unfinished due to insufficient lesson duration.	• Tasks that did not directly advance the activity's purpose (like cutting, pasting, etc.) consumed a major portion of the allotted time. • Significant delays occurred between stages. Example: During a sharing session, prolonged waits were observed because not all groups completed their tasks concurrently. • The allocated time for certain activity stages proved inadequate, hindering efficient student engagement.	• While the time assigned for most stages facilitated productivity, some were either too drawn out or rushed. Example: In an exercise about deriving a new number pattern from an existing one, ample time was provided to discern the rule. However, the subsequent phase creating a fresh pattern was hastily executed, pressuring students.	• Time assigned to every stage sufficiently bolstered overall student productivity. Transitions between stages were seamlessly executed. • Time dedicated to the activity was exclusively channelled towards achieving the mathematical learning outcome, with no wasted time on irrelevant tasks.
Key considerations for proper scoring	• When assessing segmentation and time management, it's crucial to consider both the total time allocated for the activity and the time set aside for its individual segments. • For accurate scoring, focus on how time for different stages is employed to directly support the attainment of the mathematical outcome. • During the evaluation, observe and note any time losses that may occur. These losses might not always be tied directly to the activity, but it's essential to gauge their impact on the achievement of the mathematical outcome.			

	• Examine the relevance of the time allocated to each stage in the context of the mathematical goals. Remember, time spent on foundational steps that prepare students for the final mathematical outcome can be beneficial and should not be considered a drawback in time management. • In the time allocated for various stages, pay attention to the number of students who either had to advance to the next stage without completing the current task or had to wait after finishing their work. This can be indicative of optimal time management.

Classroom management

During the implementation phase, it is essential for the teacher to actively monitor student/group behaviours either by moving around the classroom or from a dominant position. This allows for immediate interventions, when necessary, especially if certain students or groups stray from the activity. Under what we term as monitoring, teachers should navigate the classroom to ensure that student actions and efforts align with the intended task and are uninterrupted by behavioural distractions or disruptions. However, observation alone is not sufficient. When challenges arise, be they disruptions or students veering off-task, immediate and effective intervention is important. Through Intervention, teachers can employ strategies such as assisting the student in realigning their focus, adapting the content to better cater to a student's needs, or furnishing new guidelines to adjust their working method. In sum, by actively monitoring student behaviour and swiftly intervening when necessary, teachers ensure a productive and harmonious classroom environment conducive to effective learning.

Component	**Classroom management:** Involves the teacher's active monitoring of student behaviors to ensure alignment with tasks and promptly intervening with appropriate strategies when disruptions or off-task behaviours arise.			
Indicators	• **Monitoring**: Observing and ensuring that student actions and efforts are aligned with the task's objectives and are free from behavioural disruptions. • **Intervention**: Implementing appropriate strategies to guide disruptive or struggling students back to the task.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The teacher often overlooks disruptive behaviours with little or no intervention.	The teacher recognizes most disruptions but interventions often do not fully resolve them.	The teacher actively identifies disruptions and while interventions are mostly successful, some students still face brief task interruptions.	Disruptions, if present, are promptly addressed, ensuring a seamless continuation of the task for all students.

Explanations and examples	• The teacher consistently overlooks or fails to address students who disrupt the activities of others. • Example: During a competition-based activity, debates about who won overshadow the objective of the activity, and these debates are allowed to continue without resolution.	• The teacher occasionally tries to control noisy groups or students, but these interventions are often ineffective, allowing disturbances to persist. • Example: Even after being told to work quietly, a student or group continues to disrupt the classroom's harmony.	• The teacher generally detects and addresses disruptions, but there might be occasional delays in interventions, causing momentary interruptions to some students' tasks. • Example: At a certain stage of the activity, a subset of students faces localized issues. Due to the teacher's delay in noticing or intervening, these students experience interruptions in their work.	• The teacher actively monitors student behaviours and implements appropriate interventions to eliminate any localized or general issues that arise. • Example: During a group discussion, when two students begin to veer off-topic and become disruptive, the teacher promptly redirects their focus by providing them with specific questions related to the task, ensuring the smooth continuation of the activity for all students.
Key considerations for proper scoring	• When assessing the classroom management component, it's crucial to take a comprehensive view, focusing on the entirety of the activity implementation process. • If a teacher overly focuses on disruptive students/groups to the detriment of others, it may lead to receiving a lower score. It's essential for teachers to maintain a balance in their attention distribution across all students. • Activities disrupted by problematic students, regardless of whether the teacher intervenes, should not be awarded medium or high scores. A classroom's harmony is vital, and disruptions affecting the entire class should be considered seriously in the scoring process.			

Student engagement

Student engagement, at its core, encapsulates the depth and breadth of students' active involvement in activities. This involvement can be delineated into two foundational dimensions: behavioural and cognitive. Behavioural engagement revolves around the visible and tangible actions students undertake during the learning process (Skinner et al., 2009). It includes behaviours such as completing assignments, actively asking questions, and participating in collaborative teamwork within lessons. Observable participation, be it individual contributions or as part of a collective, forms the base of this engagement type. Cognitive engagement, on the other hand, delves into the realm of intellectual immersion and processing. Rather than merely participating, cognitive engagement speaks to the depth of a student's mental involvement in an activity (Danielson, 2013). It encapsulates processes such as strategy formulation, concentration, meta-cognition, and establishing logical connections. Unlike the overt nature of behavioural engagement, cognitive engagement revolves around students applying genuine intellectual effort towards achieving targeted outcomes.

With these foundations in place, we have identified three key features to assess the quality of this component: active participation, exchange of ideas and student input. Active participation stresses the essence of both behavioural and cognitive engagement. While the act of participation, such as joining discussions or team projects, speaks to behavioural involvement, the genuine intellectual effort to achieve learning objectives mirrors cognitive engagement. Exchange of ideas, here as we see it, is a blend of both engagement types. Creating an environment that fosters the sharing and collaboration among students not only promotes observable interactions but also encourages deeper cognitive processes. Students are impelled to discuss, defend their ideas, discover patterns, and logically justify their thoughts, amplifying their cognitive immersion. The third key feature is student input. Incorporating students' contributions, ranging from discussions to comments, signifies a dynamic learning landscape. Seamlessly embedding these inputs into an activity's progression not only elevates behavioral engagement but also enhances cognitive engagement. As students see their contributions shaping the trajectory of the activity, they are more likely to invest deeper intellectual energy, leading to richer cognitive outcomes.

Component	**Student Engagement:** Encapsulates the depth and breadth of students' active involvement in activities.			
Indicators	• **Active Participation**: Signifies students' genuine intellectual effort towards achieving the targeted learning outcomes. • **Exchange of Ideas**: Refers to cultivating an environment that fosters sharing and collaboration among students. • **Student Input**: Emphasizes integrating students' contributions (discussions, solutions, comments, explanations) into the progression of the activity.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Majority of students remain passive observers with minimal exchange of ideas or integration of their inputs.	Activity is driven by a select few students' participations or contributions, with little influence on achieving the targeted outcome.	While most students participate and share ideas, their collective inputs had a limited impact on the progression of targeted outcome.	Students extensively participate and share ideas, with their inputs significantly shaping the progression of the targeted outcomes.
Explanations and examples	• The activity relied solely on the teacher's knowledge transfer, without any substantial student contribution or input. • Example: The teacher perform the instruction while students passively observe without being actively engaged	• Activity primarily involved operational tasks or was steered by a subset of students, lacking in diverse idea exchange. • Examples: Students repeated common ideas rather than presenting and defending unique perspectives.	• Students engage in both operational and cognitive tasks, but the depth of idea exchange and interpretation is limited. • Examples:	• Activity saw broad and deep student involvement both in tasks and in the dynamic exchange of original ideas.

Explanations and examples		• Tasks were predominantly behavioural like cutting, pasting, or painting without meaningful cognitive engagement. • Interaction was limited to the teacher's questions and responses from a select few students.	• Students provided inputs, but these contributions were not collaboratively explored or did not significantly guide the activity toward its goal.	• Examples: Students actively worked through both operational and cognitive challenges while also sharing, interpreting, and defending their unique perspectives. • Discussions evolved with substantial student input, making their efforts pivotal in achieving the activity's targeted outcomes.
Key considerations for proper scoring	• Scoring should factor in whether the tasks and responsibilities assigned to students allow for meaningful contributions. Evaluations need to account for student contributions both throughout the activity and at various stages. • Activities where student involvement is solely limited to basic behavioral tasks (such as cutting, pasting, and painting) should not receive moderate or high scores. • Moderate to high scores are reserved for activities that promote cognitive exchanges and collaborations among students. Activities that deeply incorporate these elements, especially in the context of developing mathematical outcomes, should be scored higher. • Some activities may focus on the tasks performed by only a few students in front of the class. If there is a lack of cognitive exchange at the broader class level, such activities should receive low to very low scores for this component.			

Drawing conclusions

In this component, a critical relationship is established between the work executed by the students during the activity and the intended learning outcomes. The focus here is on achieving mathematically significant developments such as consolidation, comprehension, and development of a positive disposition. The manner in which the activity is concluded is instrumental to the overall quality of the implementation. At this stage, achieved results are actively shared with the class, establishing clear connections between the student-conducted work and the targeted learning outcomes. To ensure the activity's effectiveness, it is crucial to allocate sufficient time for this concluding phase, avoiding rushed judgments and allowing for the emergence of discoveries or insights. Various pedagogical strategies can be employed by the teacher at this juncture, including articulating the targeted learning outcomes, prompting students to express the insights and awareness they have garnered, and fostering a discussion environment conducive for the exchange of ideas and shared learnings. Two key features have been designated to evaluate the effectiveness of this component: connection and elucidation. Connection focuses on the alignment of works that students have performed during the activity with the intended learning outcome. It assesses how effectively the undertaken works reflect and lead towards the achievement of the outcome. Elucidation, on the other hand, pertains to the clarity and depth with which the learning outcomes are highlighted and discussed at the conclusion of the activity. It checks whether the wrap-up phase effectively communicates the target outcomes, prompts student reflections, and encourages sharing and discussions about the achieved results.

Component	**Drawing Conclusions:** Emphasizes the clear linkage between students' works and the intended mathematical outcomes, while ensuring these outcomes are understood and reflected upon by the students.			
Indicators	• **Connection**: Focuses on the alignment of works that students have performed during the activity with the intended learning outcome. • **Elucidation**: Pertains to the clarity and depth with which the learning outcomes are highlighted and discussed at the conclusion of the activity.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Activity concludes without drawing any conclusion, lacking connection or elucidation of the intended mathematical outcome.	Activity is concluded, but does not sufficiently emphasize the mathematical outcome or relate it to the works done.	While the works are related to the mathematical outcome, there is no verification of its clarity or how well students understood it.	The works are clearly connected with the mathematical outcome, and feedback confirms it has become clear for students.
Explanations and examples	• The activity might be prematurely concluded due to issues such as insufficient time, classroom management problems, misunderstandings regarding instructions, or ambiguities from material use. • Despite following the instructions of the activity, there was no effort to connect the works to a mathematical outcome.	• After instructions were carried out, the teacher directly conveyed the result without letting students voice their thoughts. • Students might have been allowed to share their views, but the mathematical outcome was delivered without connecting their ideas to the tasks or each other.	• Student results from the activity were utilized to elucidate the mathematical outcome, but their understanding was not verified. • Only correct student ideas were highlighted, without addressing incorrect or incomplete deductions or explaining their implications.	• There was a solid linkage between the works and the mathematical outcome, and the teacher ensured students' comprehension by checking their grasp of the target outcome.
Key considerations for proper scoring	• Activities that do not allocate sufficient time for the conclusion phase (due to class time running out or other delays) are not eligible for moderate or high scores. • For an activity to earn a moderate or high score, there should be a distinct connection between the tasks executed and the intended mathematical outcome. • Concluding insights need not be drawn only at the end; they can be made at various points throughout the activity. If conclusions are drawn at these intervals, they should be included in the evaluation. • Activities that do not engage students in reflecting on the intended results of their efforts should not qualify for moderate or high scores.			

5.2.3. Components of Mathematical Potential

The mathematical potential of the activity, designed for the evaluation of both the activity script and its implementation, is structured around three key components: depth, complexity, and focus. These components provide a comprehensive framework for assessing the mathematical richness and effectiveness of an activity and its execution. Below are brief descriptions for each of these components.

1. **Depth**: This component denotes a profound understanding of mathematical concepts and their broader applications. It refers to detailed elaboration, logical justification, and the ability to generalize knowledge to various contexts, promoting a comprehensive understanding of mathematical content.
2. **Complexity**: Complexity assesses how mathematical connections and interdisciplinary learning opportunities are organized within an activity. This component examines the complexity of relationships between mathematical concepts, solutions, and representations, as well as how different disciplines are integrated into the activity.
3. **Mathematical Focus**: This component assesses whether an activity's content is centered on mathematical development (concept, skill, thought). It examines the clarity of mathematical objectives in the activity, whether it encourages students to think mathematically, and whether it includes requirements for mathematical-focused thinking and idea generation.

In what follows, we will delve into each of these components in greater detail, including scoring criteria and indicators that offer guidance for evaluation. By grasping these components, teachers can more effectively select and design activities infused with mathematical depth. This, in turn, allows for implementations that provide students with deeper and more meaningful mathematical learning experiences.

Depth

Depth, in the context of mathematical potential, signifies the extent of understanding associated with concepts, principles, and generalizations underlying mathematical rules (Kaplan, 2017). Achieving an in-depth comprehension of the content within an activity is of paramount importance. Here, students should not only grasp the discipline's specific language but also delve deep into its intricacies. They need to elaborate on details, provide justifications for their mathematical reasoning, and effectively generalize their understanding across various contexts. These facets – elaboration, justification, and generalization – are critical in determining the depth of mathematical understanding. Elaboration pertains to the comprehensive detailing of mathematical concepts, rules, and procedures. It emphasizes the richness in explanation, providing an intricate understanding of specific mathematical content. Justification ensures that students anchor their understanding based on the core principles, laws, and generalizations that underpin mathematical rules. It covers their ability to logically explain why certain mathematical procedures are used and why they make sense. Generalization refers to the students' ability to extrapolate from specific instances, recognize trends and rules, and apply these generalizations to new and diverse situations, transcending the immediate context of learning.

Component	**Depth:** Signifies the extent of understanding associated with concepts, principles, and generalizations underlying mathematical rules			
Indicators	• **Elaboration**: Refers to the detailed understanding and thorough explanation of mathematical concepts and rules, emphasizing intricacies. • **Justification**: Covers students' capacity to ground their mathematical reasoning in core principles and laws, providing logical explanations for their ideas. • **Generalisation**: Involves the ability to extrapolate rules and trends from specific instances and apply them to various contexts.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Students solely identify or label information without deeper analysis or exploration.	Students recall basic mathematical knowledge and execute standard procedures without engaging in the underlying principles or justifications.	Students employ or apply mathematical knowledge in a manner that goes beyond simple recall but without requiring full justification or generalization.	Students deeply engage with mathematical knowledge, aiming for a through elaboration, justification, or generalization.
Explanations and examples	• Only reading or identifying data from a representation without any interpretation is required.	• Usage of mathematical rules or procedures directly, without needing interpretation or deeper understanding.	• Drawing conclusions or making inferences from provided information.	• Conducting hypothesis testing, making informed assumptions, and presenting mathematically justified evidence.

Explanations and examples	• Example: Reading sales figures for specific months from a column chart that displays these figures. Here, students are only required to identify the information presented without further interpretation.	• Example: calculation for the following expression: $2x10^2+2x10^{-2}$ In this example, students apply a direct mathematical procedure without needing to interpret or associate further.	• Making selections or comparisons from a set of alternatives based on specified criteria. • Conversion or transformation between different mathematical representations. • Example: Comparing different pricing methods (invoice, monthly payment, rent, etc.) to determine the most advantageous option. Here, students must make comparative choices based on specific criteria, going beyond simple recall.	• Formulating and explaining mathematical concepts, terminologies, and procedures relevant to a particular situation. • Interpreting and generalizing a known mathematical rule or relationship to cover new or different scenarios. • Example: Justifying the negative effects of global warming through statistical data or mathematical modelling. Students are required to interpret and generalize a rule or relationship to new situations, such as creating a mathematical justification for a real-world issue
Key considerations for proper scoring	• When scoring the "Depth" component, emphasis is placed on the verbs found in the established criteria. Assessments are made by identifying the verbs that most accurately encapsulate observed behaviours or works. The aim is to pinpoint which verbs from the scoring criteria best characterize the text or execution of the activity. • In evaluating the depth of the activity script, attention is given to the verbs that articulate the mathematical demands specified in the instructions. • Depth evaluation hinges on the end-results and the cognitive demands imposed on the students' comprehension of mathematical outcomes within the tasks at hand. • Final scoring is determined by categorizing the pertinent verbs that best describe the activity's execution under the appropriate rating criteria.			

Complexity

A central aspect of both activity design and implementation is the concept of complexity. Kaplan (2017) describes complexity as the scientific understanding of relationships formed across time, between different perspectives, and among various disciplines. The following three dimensions elaborate this definition in the context of activity design and implementation:

1. Relationships over time: Complexity is not just about immediate understanding but also about how new knowledge relates to what has been learned before. Activities should aim to help students weave a rich tapestry of understanding by connecting new insights with prior knowledge.

2. Relationships from different perspectives: Activities should be designed to encourage students to approach mathematics from multiple viewpoints. This includes, for example, finding different solutions to the same problem, recognizing the diversity of methods used to arrive at those solutions and the various ways of communicating those methods.

3. Disciplinary connections: Going beyond the strict confines of mathematics, activities should encourage an inter- or multi-disciplinary approach. This could involve making connections with daily life, technology, the arts, scientific research, economic models, historical contexts, or literary works.

Based on these dimensions, we identified two key indicators for assessing the quality of complexity in activities. The first is mathematical connections, which focuses on the student's ability to weave together mathematical concepts, solutions, and representations in a way that facilitates both the recall of past learning and the synthesis of new knowledge. The second indicator is, what we call, cross-disciplinary integration that highlights the importance of relating mathematical concepts and applications to other academic disciplines or study fields. It encourages students to apply their mathematical knowledge in diverse contexts, integrating it with insights from fields like science, economics, art and more. By giving due attention to these indicators in the design and implementation of mathematical activities, teachers can enrich students' understanding and appreciation of mathematics, allowing them to see the subject through a broader lens.

Component	Complexity: Refers to the understanding of relationships formed across time, between different perspectives, and among various disciplines			
Indicators	• **Mathematical connections:** Refers to establishing links among mathematical concepts, representations and solutions. • **Cross-disciplinary integration:** Relates mathematical ideas and applications to other academic disciplines, study fields or real-life situations.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	The activity is structured without requiring any mathematical connections.	The activity presents opportunities for connections among mathematical concepts, solutions, or representations; however, these are either used in isolation without forging links or the connections, if any, remain superficial.	The activity necessitates making connections among mathematical concepts, solutions, or representations, but interdisciplinary elements, if any, serve merely as context or setting.	The activity not only mandates mathematical connections but also enriches these with meaningful associations to other disciplines beyond mere context or setting.
Explanations and examples	• The activity remains strictly procedural, lacking both mathematical and interdisciplinary connections. • Examples: Multiple-choice exercises focused solely on operation order; tasks solely based on executing a specific formula,	• The activity introduces potential mathematical connections, but they are surface-level or unexplored. • Examples: Venn diagrams translated into lists without deeper	• While the activity emphasizes mathematical relationships, any interdisciplinary context remains as mere backdrop. • Examples: Demonstrating how the area formula for a triangle is derived	• The activity seamlessly blends intricate mathematical connections with meaningful interdisciplinary integration. • Examples: Developing a formula to predict human height based on finger length, linking real life, mathematics,

	such as the sum of consecutive numbers.	exploration; isolated tasks asking for the area of different right-angled polygons without interrelation.	from that of a rectangle; establishing relationships among percentages, decimals, and fractions within a newspaper article where the article itself does not serve any purpose beyond being a setting.	and statistics; incorporating the historical development of division algorithms into tasks; using paper-folding art (Origami) to explore geometric properties; using software tools to relate visual and algebraic representations of the concept of combination.
Key considerations for proper scoring	• Complexity puts an emphasis on how the activity connects with various concepts, solutions, representations, and domains. As such, the scoring criteria are shaped by both the quality and diversity of these connections. • To achieve a high score, the activity must meaningfully integrate mathematics with another field of study—such as science, art, technology, or daily-life. Crucially, this integration should be more than contextual; it must be decisive in the successful implementation and completion of the activity. • Activities that incorporate elements of STEM or mathematical modeling inherently include multiple disciplines or fields of study, thereby enriching the activity beyond mere contextualization. These activities are typically evaluated at the highest complexity level of 3 points. • For a moderate score (2 points), the activity doesn't necessarily need to showcase connections throughout its entirety. However, at least one segment of the activity should either necessitate or emphasize such connections.			

Mathematical Focus

The third component is Mathematical Focus. Central to this component is the necessity for the embedded and intended mathematical growth to be explicit and discernible. Upon completion of the activity, opportunities should exist for students to demonstrate development aligned with the activity's mathematical focus. In evaluating this component, we extracted two indicators. The first indicator is Mathematical Prominence. Regarding this, the activity must ensure that the mathematical aspects stand out distinctly. Among the many tasks and undertakings within the activity's scope, it is essential that mathematical components aren't overshadowed or neglected. Mathematics shouldn't be perceived as a peripheral or secondary aspect; rather, it should clearly be at the heart of the activity. The second indicator is named Mathematical Thought Provocation. The activity ought to go beyond mere task completion, delving into the realm of deep thought and idea generation. It should challenge students to align their thinking with the mathematical aspects and dimensions presented. In essence, the activity should inspire students to reflect, think critically, and generate ideas aligned with its mathematical objectives.

It is crucial to ensure that the activity is not just going through the motions but actively engaging students in the targeted mathematical endeavours. It is not enough for students to merely complete tasks; they must be actively engaged, ensuring they delve deeply into the targeted mathematical concepts and objectives.

Component	**Mathematical Focus:** Refers to the clear and central emphasis of an activity on specific mathematical concepts or skills.			
Indicators	• **Mathematical Prominence:** Refers to the centrality and distinguishability of the mathematical objectives within an activity. • **Mathematical Thought Provocation:** A assesses the extent to which an activity encourages idea generation specifically aligned with its mathematical objectives.			
Score	**Very Low (0)**	**Low (1)**	**Moderate (2)**	**High (3)**
Criteria	Mathematical objectives are unclear or indistinguishable, and tasks do not provoke thought or reflection on them.	Mathematical objectives are distinguishable, but tasks do not actively provoke thought or reflection on them.	Mathematical objectives are distinguishable; tasks provoke thought and reflection but give overt hints towards the desired outcome.	Mathematical objectives are distinguishable, and tasks effectively provoke critical thought and idea generation aligned with them.

Explanations and examples	• Many operational tasks (such as cutting, pasting) are carried out within the scope of the activity, but the targeted mathematical focus remains unclear after these works. • Although there is a structured implementation in the activity, the intended mathematical outcome following these steps is ambiguous. For instance, playing a card game without a clear understanding of its mathematical focus or objective.	• Tasks do not require any reflective thought concerning the actions taken during the activity. • Students follow instructions in a step-by-step manner and carry out certain operations, yet there's no directive that prompts them to contemplate the mathematical focus. An example would be using math software to draw geometric shapes based solely on procedural steps, without understanding or reflecting on the mathematical significance.	• The desired outcome or understanding is made overtly clear in the instructions given to the students, leaving little room for independent discovery or idea generation. For instance, rather than asking students to "identify the relationship between the areas of the triangle and the rectangle", they might be prompted with leading questions like, "Do you notice that the triangle's area is half of the rectangle's?"	• Tasks are constructed in a way that mandates students to deliberate on the mathematical outcome, or that spurs reflective thinking on their actions. For instance, asking students to "identify the criteria for a shape to be classified as a polygon" after engaging with relevant tasks ensures that students are aligning their actions with the mathematical objectives.

Key considerations for proper scoring	• "Mathematical focus" denotes the specific mathematical development of any kind, such as concept formation and skill acquisition, that an activity and its tasks aim to cultivate. In this respect, the scoring under this component evaluates the clarity or identifiability of the target mathematical outcome inherent in the activity. • During the scoring process, tasks within the activity that encourage mathematical thinking should be designated and considered. • For the assessment of this component, both moderate and high scores necessitate tasks that prompt students to reflect upon the mathematical outcome. However, for a moderate score, the tasks explicitly guide students towards the desired outcome. In contrast, a high score implies that students are encouraged to generate ideas around the outcome without overt guidance.

CHAPTER 6

CONSIDERATIONS IN USING F*f*MA

FfMA was crafted to offer a comprehensive perspective on activity-based mathematics instruction. Its dimensions and components aim to assess the quality of both the activity script and its implementation, focusing solely on observable attributes. Thus, FfMA is specifically engineered to rate the quality of discernible aspects that are pivotal in the design and implementation of activities in mathematics instruction.

Assessments conducted using FfMA have the potential to significantly enhance the quality of activity-based mathematics instruction. By considering specific classroom conditions and student needs, the utilization of FfMA helps to ensure that the adaptation process is guided by rational decision-making rather than intuition. This lends a systematic and rigorous approach to instructional design and execution. Taking into account the components and scoring criteria of FfMA is crucial for the successful simulation or mental rehearsal of how the activity script will play out in the classroom, both before and after adjustments are made. Conducting the simulation process with FfMA's implementation in mind helps anticipate potential challenges. Furthermore, it offers guidance to teachers on strategies and measures to mitigate any factors that might negatively impact the execution of the activity.

The adaptation and simulation cycle is iterative, continuing until the teacher is confident that the activity will be successfully implemented. During each iteration, the teacher makes strategic modifications for improvement, focusing on adaptation and/or revising the activity script. Upon completing this cycle, the activity is finalized for classroom implementation. While this process may appear complex at first glance, it has been observed that as teachers gain experience with FfMA, they are better able to navigate the adaptation-simulation cycle in a more rational and efficient manner.

Two additional key elements contribute to enhancing the quality of a mathematical activity designed through the adaptation-simulation process and evaluated by FfMA: reflection and revision. These elements emphasize that activity-based learning is a continually evolving practice. The reflection process relies on observations made during the activity's implementation, as well as insights garnered from a post-implementation evaluation based on FfMA. By focusing on the various components and scoring criteria within FfMA's implementation dimension, this evaluation yields valuable information for reflective thinking. It also offers teachers constructive feedback about the efficacy of the implemented activity and identifies areas requiring improvement.

The revision process commences as the teacher makes modifications to the activity design, which naturally affects the selection of activities. These changes are introduced in light of the strengths and weaknesses identified during the implementation stage. In fact, any changes deemed necessary by the teacher following a reflective evaluation fall under the purview of the revision process. This process aids teachers in adapting the same activity for use in other classes or in subsequent academic years. Utilizing FfMA for reflection and revision contributes not only to teachers' professional development in effectively implementing activity-based instruction but also serves to enhance their expertise in this educational approach.

For the effective application of FfMA, it is crucial that evaluators or raters possess a profound understanding of mathematics and have experience in mathematics education. Evaluators lacking a foundational knowledge of mathematics or unfamiliar with the fundamental concepts of mathematics education may produce inconsistent scores when using FfMA. In addition to this prerequisite knowledge, there are other considerations to bear in mind when utilizing FfMA for assessment and feedback. These considerations are discussed further in this section.

6.1. Considerations in Using F*f*MA as an Evaluation Tool

FfMA is specifically designed to assess the quality of activity design and its corresponding implementation within the context of mathematics education. It is not intended to serve as a general evaluation tool for a mathematics course as a whole. Rather, its focus is narrowed to scrutinizing the quality of the activity script and the execution of that script in a classroom setting. With that in mind, it is important to highlight certain considerations pertinent to the evaluation procedures that rely on FfMA.

- FfMA is a versatile tool designed to assess both the activity script and its implementation, either together or separately. Nevertheless, when conducting an assessment of both the activity script and its implementation, it is advisable to evaluate the script prior to its implementation. Doing so minimizes the potential bias wherein the implementation influences the evaluation of the script itself.
- Activity implementation might span a specific segment of a mathematics lesson (often within a constrained timeframe), or it might be conceived to stretch over multiple sessions. As an assessment instrument, FfMA concentrates on the full execution of the activity, limited to the continuum from its commencement to its conclusion.
- Efforts related to mathematics instruction before and after the activity execution are not considered in the evaluation using FfMA.
- FfMA can serve as an in-class observation tool for external reviewers as well. In that case, for optimal utilization of FfMA, capturing a recording of the implementation and then conducting an assessment by the evaluator will contribute to generating more valid and reliable results.
- When considering evaluation criteria, one should refer to the explanations provided in FfMA, keeping in mind the provided examples and key considerations. To maintain the validity and reliability of FfMA's use, the evaluator must thoroughly examine the dimensions and components within FfMA and become acquainted with it by assessing activity scripts and recordings of activity-based implementations conducted in real settings. Hence, gaining experience in using FfMA is crucial for ensuring consistent evaluations by the scorer.
- While FfMA has the capability to generate a cumulative score as an analytical rubric, the primary emphasis should be on the components with lower scores. Efforts should be directed towards enhancing the quality in these areas, with interventions or modifications introduced to improve scores in subsequent implementations.
- Evaluators, be they teachers or scorers, should not aim for either low or high scores when assessing the activity script and the quality of the implementation. The assessment should be conducted objectively, referencing the indicators and scoring criteria, as well as any provided explanations and examples. Coding should align with the score that most accurately reflects the characteristic being evaluated (FfMA component).

6.2. Considerations in Using F*f*MA as a Feedback Tool

Feedback, often described as information given about specific behaviours and practices, holds a crucial role in the learning process (Wiggins, 2012). It is conceptualized as the information furnished by one individual (such as a teacher or a peer) to another about their performance or comprehension (Hattie & Timperley, 2007). Feedback serves as a significant variable that can enhance an individual's development, motivation (Lam et al., 2011, p.218), and overall performance (Nelson & Schunn, 2009).

This concept is a key element of effective learning as it offers guidance on how to refine and improve a performance. Hattie (1999) considers feedback as "the strongest moderator that increases success" in educational settings. Therefore, the provision of constructive feedback is not merely supplementary but rather integral to enhancing the quality and effectiveness of teaching and learning experiences.

When using FfMA as a feedback tool for assessing the quality of the activity script and implementation, the following considerations should be taken into account:

- When conducting evaluations using FfMA, a particular emphasis should be placed on the component(s) that received low scores. The teacher should delve into understanding the underlying reasons for such scores. Based on these insights and through reflective consideration, efforts should be directed towards enhancing the quality of subsequent implementations. Appropriate measures and modifications should be introduced to improve scores in future applications.
- In evaluations conducted based on FfMA concerning the activity script, the design should be refined through adaptation and simulation processes. Preparation for the actual implementation should consider the components specifically related to the execution phase of the activity.
- Scores generated for each component via FfMA should be analyzed through reflective thinking, leading to appropriate revisions aimed at enhancing the quality of the design. These adjustments should also be incorporated into the practical execution of the activity. This will facilitate improvements in subsequent iterations of the implementation. When repetition is not feasible, strategies to address the shortcomings related to components that received low scores should be considered, ensuring the efficacy of future implementations is optimized.

CHAPTER 7

CONTRIBUTION OF FfMA TO THE FIELD OF MATHEMATICS EDUCATION

In this chapter, we discuss the theoretical and practical contributions of FfMA to the field of mathematics education. Initially, we attend to the significance of FfMA, in light of relevant literature. To elucidate its practical relevance, we identify potential users and describe the application of this tool in their context. The chapter concludes with suggestions for subsequent research related to the further utilization of FfMA.

7.1. Importance of F*f*MA

When reviewing the existing literature, it becomes evident that there is a notable absence of comprehensive studies that offer a holistic evaluation of mathematical activity scripts and their implementations. The existing evaluations in these studies tend to be narrow in scope, often focusing on specific variables or delving deeply into selected features for research purposes. For instance, Lozano (2017) examined the restrictiveness of instructions, while Glassmeyer (2019) concentrated on students' working styles and sharing principles. It is important to note that these studies, and the likes, tend to zoom in on very specific characteristics, providing a rather limited perspective. Consequently, they fall short of presenting the comprehensive view necessary for assessing the quality of the activity-based instruction process. What sets FfMA apart from other assessment approaches in the field is its unique ability to offer a comprehensive and holistic perspective by structuring activity-based mathematics instruction through design and implementation.

The FfMA offers a platform to evaluate and furnish feedback on the activity script, the process of its implementation, and the mathematical potential intrinsic to both dimensions. While there is a dearth of empirical research on this topic, a few studies, such as the one by Güzel (2020), have endeavoured to holistically explore activity design teaching by considering its various dimensions. However, these studies often fall short in terms of thoroughly analysing the activity-based teaching process across its interconnected dimensions or components.

Furthermore, they seem to lack comprehensive research on essential performance indicators and the nuanced grading criteria vital for a sound quality assessment. Given this, FfMA emerges as a promising tool designed to bridge this evident gap.

When examining existing research on the quality of mathematical activity scripts and their implementation, it becomes apparent that evaluations typically centre around either content-specific or pedagogical features. For instance, Stein et al. (1996) focused on the cognitive demands of activities, while Clarke and Roche (2018) emphasized content-related aspects such as the activity's context. Bozkurt (2018) examined the activity texts in terms of purpose, student cooperation and applicability. In a different vein, Özmantar and Bingölbali (2009) addressed the pedagogical dimensions, such as the use of teaching materials and classroom management strategies. Contrary to other models that focus solely on either pedagogical approach or mathematical content, FfMA uniquely integrates both of these aspects into its assessment framework. This dual focus enables FfMA to offer a more comprehensive evaluation of the quality of activity scripts and their subsequent implementation. Such a comprehensive approach allows FfMA to offer a more nuanced evaluation, enriching the overall quality of both the design and the implementation of mathematical activities.

In the literature, there are several studies that propose principles referring to the quality of activity scripts and certain facets of their implementation (e.g., Özmantar & Bingölbali, 2009; Yeşildere İmre, 2020). Also in the literature studies evaluating the activity texts based on these principles (e.g. Kerpiç & Bozkurt, 2011). However, these works largely rely on literature reviews rather than empirical evidence. As a result, the practical and theoretical merits of the principles suggested for design and implementation lack evidence-based validation. In contrast, FfMA stands out by offering components, indicators, and scoring criteria—collectively referred to as "design principles"—that are grounded in empirical insights, thus filling the evidentiary gap left by previous studies.

Within the scope of research on mathematical activities, mathematics is frequently viewed as a mere context instead of being treated as the central essence of the activity. Despite many studies being anchored in mathematical instruction, the inherent mathematical core of these activities often gets overshadowed by more general educational themes. This is evident in the works of Coles and Brown (2016), who delve into students' roles in activities, or Watson and Mason (2007), who spotlight classroom dynamics. Similarly, Komatsu and Jones (2019) prioritize the concept of "purposefulness" in their analysis. As a result, the intrinsic mathematical nature is often relegated to the periphery in these scholarly discussions.

Contrastingly, FfMA emphasizes the need to bring mathematics to the forefront of activity design and implementation. It advocates for the evaluation of the mathematical quality of the intended outcomes. Therefore, FfMA considers the mathematical potential of both the activity script and its implementation as an integrated evaluation dimension. This approach sends a distinct message to both practitioners and researchers: the design and implementation of activities should not just cater to pedagogical, psychological, and affective aspects, but should also prioritize the mathematical characteristics that make these activities unique to the field of mathematics education.

The "Mathematical Activity Design and Implementation Model," which serves as the foundational framework for FfMA, offers practical utility by holistically addressing all facets of the activity-based instruction process. It delineates both the design and implementation phases as well as the transitional elements that connect them. This framework uncovers the dynamic and dialectical nature of the activity script-design-implementation sequence, particularly in relation to phenomena of adaptation-simulation and reflection-revision. This insight is crucial for comprehensively understanding the activity-based instruction process, identifying the aspects that warrant evaluative focus, and recognizing stages where opportunities for development and improvement are inherent.

Based on the Mathematical Activity Design and Implementation Model, the FfMA has been structured to comprehensively delineate the attributes essential for evaluating the quality of activity text and implementation processes, pivotal for activity-based instruction. This entails detailed dimensioning of features, grading of potential performance levels, and formulation of pertinent criteria. Both the model and FfMA have been crafted in response to the shortcomings observed in existing research focused on activity design and implementation, thereby aiming to bridge the identified gaps.

One distinctive feature of FfMA lies in its developmental methodology, specifically its employment of design-based research to create both the Mathematical Activity Design and Implementation Model as well as the FfMA itself. This approach was spurred by a recognized gap between theory and practice in the field of education (diSessa & Cobb, 2004). The dilemma extends to studies concerning activity-based learning, wherein the real-world application of scientific knowledge remains markedly limited. Design-based research has gained considerable traction since the early 2000s as a method to bridge this divide (Kelly, 2003; Sandoval & Bell, 2004). It serves a functional role in fostering products with practical educational value. The methodology is inherently collaborative, involving both researchers and practitioners to identify problems and develop solutions. It is characterized by iterative cycles of testing and refining products in real-world settings, with the ultimate aim of generating principles for solutions-oriented practice (Reeves, 2006). Therefore, FfMA's design-based research roots underscore its commitment to not just theoretical rigor but also practical utility and implementation, ensuring that it remains both theoretically sound and pragmatically applicable.

The development of FfMA hinged on the design-based research methodology, as mentioned above. The introduction of a tool designed to evaluate activities and offer feedback to practitioners is somewhat pioneering in the studies focusing on instructional activities. In its formation, FfMA seamlessly integrated insights from field specialists, active practitioners, researchers, and empirically-grounded knowledge. Practitioners played a critical role in assessing FfMA's practical utility. Their feedback steered iterative refinements that defined its final version. The culmination of this process is a robust, evidence-based tool designed to guide teachers, evaluate their efforts, and provide feedback on activity design and implementation. Given the novel nature of its developmental approach, the methodology underpinning EDGA can certainly be considered original.

FfMA presents both a theoretical and conceptual framework that holds value for researchers in this specialized domain. Its utility lies in its dual nature: while serving practical ends by focusing on observable features to gauge implementation quality, it also introduces an underlying model that has theoretical implications. This synergy between theory and practice is a critical aspect that design-based researchers prioritize (Andriessen, 2007). Products developed through this methodology not only enhance the practical utility of field-specific studies but also enrich them theoretically, as they are informed by real-world practice. This allows for meaningful comparison of results and furthers the advancement of scientific understanding in the domain. Therefore, FfMA is expected to make substantive theoretical and conceptual contributions to research in activity-based mathematics education.

7.2. Potential Users of F*f*MA

FfMA serves as a versatile tool designed for multiple audiences, catering to both research and practical needs. It is beneficial for evaluating activity scripts and their implementation, thereby offering valuable insights for researchers. Teachers can also utilize FfMA as a resource to refine and enhance their instructional practices. Its potential users encompass mathematics teachers and candidates as well as researchers in mathematics education. Furthermore, those who are involved in teacher training, as well as those involved in crafting activities for digital or printed resources, stand to gain from the utility of FfMA. Recommendations tailored for these specific user groups are outlined below.

Mathematics teachers: Mathematics teachers represent the primary intended users of FfMA. This tool is especially valuable for those who incorporate activities into their mathematics instruction. FfMA offers guidance on both the design and successful execution of activities tailored for mathematics instruction. By enhancing their skills in activity design and implementation, teachers can further optimize the efficacy of their classroom practices. Moreover, FfMA serves as an instrumental self-assessment resource, contributing to the continuous professional growth for teachers.

Pre-service mathematics teachers: FfMA is anticipated to play a pivotal role in equipping pre-service mathematics teachers with skills in activity-based instruction, enhancing their proficiency in both activity design and implementation. Teacher educators can integrate FfMA into their method courses, providing prospective teachers with a tangible resource to draw upon. This very book was prepared to guide the potential users who could hence delve into the theoretical underpinnings of FfMA and gain insights into its application. This guidebook is envisaged to augment the practical incorporation of FfMA into mathematics teacher education programs.

Researchers and academics: Beyond its practical significance, FfMA also offers a theoretical contribution by presenting a framework adaptable to academic pursuits. Envisioned as a theoretical compass, FfMA has the potential to steer fresh research endeavours, serving as an inspirational source for scholars when proposing new research projects.

Teacher educators: FfMA presents opportunities for professional growth, facilitating the creation of optimal learning environments and enhancing activity procedures, allowing participants to take charge of their own learning. Accordingly, teacher educators involved in training sessions can effectively utilize FfMA.

Activity developers: For those involved in crafting activities for textbooks and other printed or digital materials, FfMA is an invaluable tool. With its solid theoretical grounding coupled with practical utility, FfMA is expected to assist in designing activities that resonate effectively in actual classroom settings.

7.3. Recommendations for Further Research

FfMA establishes distinct performance indicators in the context of activity-based mathematics instruction. It serves as a guiding framework for teachers to structure student activities around mathematical content effectively. Moreover, FfMA provides a foundation for the evaluation of activities that go beyond the curriculum of a single subject matter, such as those designed for STEM or interdisciplinary works. This makes it a valuable resource for future research aimed at developing rubrics or other evaluative metrics in different subject matter areas.

Activities can be categorized based on their location of execution: those confined to the classroom, those conducted outside the classroom, and those adaptable for both settings (Wassermann et al., 2007). FfMA is versatile, catering to the evaluation of all these types of activities. Furthermore, with the growing emphasis on digital education, FfMA also holds promise for assessing online activities that are becoming increasingly prevalent in current educational practices.

FfMA shows promising potential for generating meaningful outcomes in subsequent research, particularly in correlational studies. For instance, examining the relationship between total scores derived from FfMA and the permanence of learning can further enrich our understanding of activity-based learning.

Further research could explore the applicability of FfMA, originally developed within the context of secondary school mathematics, for assessing the quality of activity-based instructional practices at primary, high school, and undergraduate levels.

We believe that FfMA holds the potential to offer insights and guidance for developing similar tools to assess the quality of activity-based teaching practices in subjects beyond mathematics, such as Science, Turkish, and Social Studies.

Conducting longitudinal studies could enable both the tracking of teachers' progression under FfMA's guidance and a deeper comprehension of FfMA's role. Moreover, by juxtaposing FfMA with different evaluation frameworks, a more holistic understanding of its effectiveness can be attained.

In this comprehensive exploration of the FfMA, we have delved deep into its intricacies, implications, and potential applications. From its foundational principles to its potential use in both classroom and research settings, FfMA emerges as a versatile tool. Its potential to reshape activity-based instruction in mathematics, offering guidance for teachers, pre-service teachers, activity designers, and researchers alike, has been highlighted. As we culminate this book, it is evident that FfMA not only bridges the gap between theoretical knowledge and practical application but also paves the way for future research and innovations in mathematics education. We hope that researchers and practitioners find this resource insightful and instrumental in enhancing the quality and impact of mathematical activities.

REFERENCES

Agyei, D. D., & Voogt, J. M. (2016). Pre-service mathematics teachers' learning and teaching of activity-based lessons supported with spreadsheets. *Technology, pedagogy and education, 25*(1), 39-59.

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, *32*(1), 23-38.

Andriessen, D. (2007). Designing and testing an OD intervention: reporting intellectual capital to develop organizations. *The Journal of Applied Behavioral Science*, *43*(1), 89-107.

Artigue, M. (2009). *Didactical design in mathematics education*. In C. Winslow (Ed.), Nordic research in mathematics education: Proceedings from NORMA08 in Copenhagen (pp. 7–16). Rotterdam: Sense Publishers.

Baki, A. (2008). Mathematics education from theory to practice, 4th Edition, Ankara: Letter Education Publishing.

Baki, A., Yalçınkaya, H., Özpınar, İ., & Uzun, S. (2009). Comparing Views of Primary School Mathematics Teachers and Prospective Mathematics Teachers about Instructional Technologies. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *1*(1), 65-83.

Barrett, J. E., & Battista, M. T. (2014). *Two approaches to describing the development of students' reasoning about length: A case study for coordinating related trajectories Learning over time,* Learning trajectories in mathematics education (pp. 97-124). Charlotte, NC: Information Age Publishing.

Baykul, Y. (2000). *Measurement in education and psychology: Classical test theory and application*, Ankara: OSYM press.

Becker, J. P., & Shimada, S. (1997). *The open-ended approach: A new proposal for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Boaler, J. (1993). The Role of Contexts in the Mathematics Classroom: Do they Make Mathematics More" Real"?. *For the learning of mathematics*, *13*(2), 12-17.

Bozkurt, A. (2012). Mathematics teachers' perceptions of mathematical activities. *Education and Sciences, 37*(166). 101-115.

Bozkurt, A. (2018). An evaluation of the 6th grade mathematics textbook in terms of aim, student work format and applicability *Electronic Journal of Social Sciences, 17*(66), 535-548.

Bransford, J.D., & Donovan, S.M. (2005). Scientific inquiry and how people learn. In S.M. Donovan & J.D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 397– 420). Washington, DC: The National Academies Press.

Brousseau, G. (1998). *Theory of didactical situations in mathematics*. Dordrecht, The Netherlands: Springer.

Buchheit, M., Spencer, M., & Ahmaidi, S. (2010). Reliability, usefulness, and validity of a repeated sprint and jump ability test. *International Journal of Sports Physiology and Performance*, *5*(1), 3-17.

Cameron, T., & Bennett, S. (2010). Learning objects in practice: The integration of reusable learning objects in primary education. *British Journal of Educational Technology, 41*(6), 897-908.

Choy, B. H. (2016). Snapshots of mathematics teacher noticing during task design. *Mathematics Education Research Journal*, *28*(3), 421-440.

Chun Tie, Y., Birks, M., & Francis, K. (2019). Grounded theory research: A design framework for novice researchers. *SAGE open medicine*, *7*, 2050312118822927.

Clare, L. (2000). Using Teachers' Assignments as an Indicator of Classroom Practice. *CSE Technical Report.*

Clarke, D., & Roche, A. (2018). Using contextualized tasks to engage students in meaningful and worthwhile mathematics learning. *The Journal of Mathematical Behavior*, *51*, 95-108.

Coles, A., & Brown, L. (2016). Task design for ways of working: making distinctions in teaching and learning mathematics. *Journal of Mathematics Teacher Education*, *19*(2), 149-168.

Cornwall, J., & Graham-Matheson, L. (2011). *Leading on Inclusion: Dilemmas, Debates, and New Perspectives*. Florence, KY: Routledge.

Crocker, L., & Algina, J. (1986). *Introduction to classical and modern test theory*. Holt, Rinehart and Winston, 6277 Sea Harbor Drive, Orlando, FL 32887.

Danielson, C. (2013). *The Framework for Teaching Evaluation Instrument*. The Danielson Group.

diSessa, A. A. & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences*, 13(1), 77–103.

Doerr, H. M. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics*, 62, 3–24.

Doig, B., Groves, S., & Fujii, T. (2011). The critical role of task development in Lesson study. In L. Hart, A. Alston & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 181–199). Dordrecht, The Netherlands: Springer.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational psychologist*, 23(2), 167-180.

Doyle, W., & Carter, K. (1984). Academic tasks in classrooms. *Curriculum Inquiry*, 14, 129–149.

Driver, R., & Oldham, V. (1986). A constructivist approach to curriculum development in science. *Studies in Science Education,* 13, 107-112.

Erdem, Ö., & Aktaş, G. S. (2018). Assessment of activity-based instruction in overcoming 7th grade middle school students' misconceptions in algebra. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *9*(2), 312-338.

Ferguson, S. (2009). *Teachers' Use of Mathematics Tasks: The Impact on the Mathematics Learning and Affective Responses of Low-attaining Upper Primary Students*. Crossing Divides: Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia 1(1), 185–192.

Fitriati, F., & Novita, R. (2018). *Designing student worksheet for rich mathematical tasks.* In Journal of Physics: Conference Series (Vol. 1088, No. 1, p. 012029). IOP Publishing.

Fitriati, F., Marlaini, M., & Elizar, E. (2021). Integrating Rich Task into the Mathematics Classroom to Develop Students' Higher Order Thinking Skills: A Collaborative Action Research Study in a Secondary School. *Elementary Education Online Online*, 20(1). 479-494

Fitriati, F., Novita, R., & Johar, R. (2020). Exploring the usefulness of rich mathematical tasks to enhance students' reflective thinking, *Cakrawala Pendidikan*, 39 (2), 346-389.

Foster, C. (2017). Developing Mathematical Fluency: Comparing exercises and rich task. *Educational Studies in Mathematics*, 1-21.

Glassmeyer, D. (2019). Developing Mathematics Teachers' Attention to Quantitative Reasoning in Task Design: A Modeling Approach. *Numeracy: Advancing Education in Quantitative Literacy*, *12*(1).

Glover, A. (2016). Exploring Misconception with Rich Tasks. *Ohio Journal of School Mathematics.* 73. 43-49. http://hdl.handle.net/1811/80137.

Goos, M., Geiger, V., & Dole, S. (2013). Designing rich numeracy tasks. *ICMI Study 22: Task Design in Mathematics*, 589-597.

Gömleksiz, M. N. (2005). Evaluation of the effectiveness of the new primary education curriculum in practice. *Educational Sciences: Theory and Practice*, 5(2), 339–370.

Griffin, P. (2009). What Makes a Rich Task?. *Mathematics teaching*, 212, 32-34.

Gürbüz, R., Çatlioğlu, H., Birgin, O., & Erdem, E. (2010). An Investigation of Fifth Grade Students' Conceptual Development of Probability through Activity Based Instruction: A Quasi-Experimental Study. *Educational Sciences: Theory and Practice*, *10*(2), 1053-1068.

Güzel, B., & Alkan, H. (2005). Evaluating pilot study of reconstructed Turkish elementary school curriculum. *Educational Sciences: Theory & Practice 5*(2), November, pp. 410-420.

Güzel, M. (2020). *A model proposal to evaluate design and implementation quality of mathematical learning activities.* Unpublished doctoral thesis. Gaziantep University, Institute of Education Sciences, Gaziantep, Türkiye.

Güzel, M., Bozkurt, A., & Özmantar, M.F. (2021). Analysis of the evaluations and interventions made by mathematics teachers in the activity design and implementation process. *Journal of Computer and Education Research*, *9*(18), 513-545.

Güzel, M., Bozkurt, A., & Özmantar, M.F. (2020). An Investigation of Text Book Activities' Purposes from the Perspective of Mathematics Teachers, *Cumhuriyet International Journal of Education,* 9(3), 875-896.

Hattie, J. A. (1999). Influences on student learning (*Inaugural professorial address, University of Auckland, New Zealand*). Retrieved from http://www.arts .auckland.ac.nz/ staff/index.cfm?P=8650

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research*, *77*(1), 81-112.

Henningsen, M., & Stein, M. K., (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524–549.

Herbst, P. (2008). The teacher and the task. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education 1*. 125–131. Morelia: PME.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and instruction*, *26*(4), 430-511.

Horoks, J., & Robert, A. (2007). Tasks designed to highlight task-activity relationships. *Journal of Mathematics Teacher Education*, *10*(4-6), 279-287.

Hugener, I., Pauli, C., Reusser, K., Lipowsky, F., Rakoczy, K., & Klieme, E. (2009). Teaching patterns and learning quality in Swiss and German mathematics lessons. *Learning and Instruction*, 19(1), 66–78.

Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-36).

Jones, K., & Pepin, B. (2016). Research on mathematics teachers as partners in task design. *Journal of Mathematics Teacher Education*, *19*(2-3), 105-121.

Jonsson, A., & Svingby, G. (2007). The use of scoring rubrics: Reliability, validity and educational consequences. *Educational Research Review, 2*, 130-144.

Kaplan, S. N. (2017). Differentiating with depth and complexity. *In Fundamentals of Gifted Education* (pp. 270-278). Routledge.

Kelly, A. E. (2003). Research as design. *Educational researcher*, *32*(1), 3.

Kelly, A. E. (2016). Design research in education: Yes, but is it methodological?. In Barab, S.A., & Squire, K. (Eds) *Design-Based Research: Clarifying the Terms* (pp. 115-128). Psychology Press.

Kerpiç, A., & Bozkurt, A. (2011). An evaluation of the 7th grade mathematics textbook tasks within the framework of principles of task design. *Journal of Mustafa Kemal University Institute of Social Sciences, 8*(16), 303-318.

Keser Özmantar, Z. (2018). Design Based Research, (Eds. Kadir Beycioğlu, Niyazi Özer, Yaşar Kondakçı), *Research in Educational Administration,* 441 - 460, Ankara: Pegem Academy.

Kieran, C., Doorman, M., & Ohtani, M. (2015). Frameworks and principles for task design. In *Task Design in Mathematics Education* (p.19-81). Springer International Publishing.

Komatsu, K., & Jones, K. (2019). Task design principles for heuristic refutation in dynamic geometry environments. *International Journal of Science and Mathematics Education*, *17*(4), 801-824.

Lam, C. F., DeRue, D. S., Karam, E. P., & Hollenbeck, J. R. (2011). The impact of feedback frequency on learning and task performance: Challenging the "more is better" assumption. *Organizational Behavior and Human Decision Processes*, *116*(2), 217-228.

Liljedahl, P., Chernoff, E., & Zazkis, R. (2007). Interweaving mathematics and pedagogy in task design: A tale of one task. *Journal of Mathematics Teacher Education, 10*(4-6), 239-249.

Lozano, M. D. (2017). Investigating task design, classroom culture and mathematics learning: an enactivist approach. *ZDM*, *49*(6), 895-907.

Margolinas, C. (2013). Task design in mathematics education. P*roceedings of ICMI study 22. In ICMI Study 22.*

Martin, A. (2003). Dominoes. *Mathematics in School*, 32(4), 41.

Mason, J. (2020). Generating worthwhile mathematical tasks in order to sustain and develop mathematical thinking. *Sustainability*, 12(14), 5727.

Mason, J., & Johnston-Wilder, S. (2004). *Fundamental constructs in mathematics education.* London: Routledge Falmer.

MEB, (2018), *6th Grade Mathematics Textbook. Ankara: State Books*

Molloy, E., & Boud, D. (2012). Changing conceptions of feedback. In *Feedback in higher and professional education* (pp. 11-33). Routledge.

Molloy, V. E. (2013). *Boneyard, an original novella, accompanied by an extended essay on experimental narrative strategies in selected examples of contemporary fiction*, University of KwaZulu-Natal Doctoral dissertation, South Africa.

Moskal, B. M., & Leydens, J. A. (2000). Scoring rubric development: Validity and reliability. *Practical Assessment, Research & Evaluation, 7*(10), 71-81.

Nelson, M. M., & Schunn, C. D. (2009). The nature of feedback: How different types of peer feedback affect writing performance. *Instructional science*, *37*(4), 375-401.

Nyman, R. (2016). What makes a mathematical task interesting?. *Educational Research and Reviews*, *11*(16), 1509-1520. DOI: 10.5897/ERR2016.2919

Olkun, S., & Toluk, Z. (2005). *Activity-based mathematics teaching in primary education*. Ankara: Ani Publishing.

Özgen, K. (2017). A theoretical study on mathematical learning activities: a case of function concept, *Abant İzzet Baysal University Journal of the Faculty of Education*, *17*(3), 1437-1464.

Özmantar, M. F., Bozkurt, A., Demir, S., Bingölbali, E., & Açıl, E. (2010). Elementary teachers' perception of instructional activities. *Journal of Selcuk University Ahmet Keleşoğlu Education Faculty*, *30*, 379-398.

Özmantar, M.F., & Bingölbali, E. (2009) Activity design and basic design principles. In Bingölbali, E., Özmantar, M.F. (Ed), *Mathematical Challenges Encountered in Primary Education and Suggestions for Solutions*. Pegem Academy, Ankara.

Öztürk, F., & Işık, A. (2020). Investigation of the Perceptions of Prospective Primary Mathematics Teachers on the Concept of Activity. *Celal Bayar University Journal of Social Sciences, 18 (1), 47-63.*

Piggott, J (2012). *Rich Task and Contexts*, http://nrich.maths.org/5662. Citation date: 21.08.2022.

Pokhrel, T. R. (2018). Activity based mathematics instruction: Experiences in addressing the 21st-century skills. *Journal of Mathematics Education*, *11*(1), 46-61.

Reeves, T. (2006). Design research from a technology perspective. In *Educational design research* (pp. 64-78). Routledge.

Revina, S., & Leung, F. K. S. (2019). How the same flowers grow in different Soils? The implementation of realistic mathematics education in Utrecht and Jakarta classrooms. *International Journal of Science and Mathematics Education*, *17*(3), 565-589.

Rogers, C., & Freiberg, H. J. (1994). *Freedom to learn*. OH: Merrill.

Sadler, D. R. (2009). Indeterminacy in the use of preset criteria for assessment and grading. *Assessment & Evaluation in Higher Education*, 34(2), 159-179.

Sadler, R. (1989). Formative assessment and the design of instructional systems. *Instructional Science, 18*, 119-144.

Sandoval, W. A., & Bell, P. (2004). Design-based research methods for studying learning in context: Introduction. *Educational psychologist*, *39*(4), 199-201.

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM*, *45*(4), 607-621.

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical thinking and learning*, *6*(2), 91-104.

Skinner, E. A., Kindermann, T. A., & Furrer, C. J. (2009). A motivational perspective on engagement and disaffection: Conceptualization and assessment of children's behavioral and emotional participation in academic activities in the classroom. *Educational and Psychological Measurement*, *69*(3), 493-525.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, 3(5), 344–350.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, *3*(4), 268-275.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455–488.

Suzuki, K., & Harnisch, D. L. (1995). Measuring Cognitive Complexity: An Analysis of Performance-Based Assessment in Mathematics. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, California [ERIC document: Ed390924].

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education, 10*(4-6), 217-237.

Toprak, Ç., Uğurel, I., & Tuncer, G. (2014). Analysis of Math Learning Activities Developed By Pre-service Teachers in Terms of the Components of Content, Purpose, Application Methods. *Turkish Journal of Computer and Mathematics Education* (TURCOMAT), 5(1), 39-59.

Uğurel, I., & Bukova-Güzel, E. (2010). A discussion on mathematics learning tasks and a conceptual framework proposal. *Hacettepe Universty Journal of Education, 39*, 333-347.

Üstündağ Pektaş, Y. (2018) Secondary School Mathematics 8th Grade Textbook. Ankara: Ogun Press.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic mathematics education. *Encyclopedia of mathematics education*, 713-717.

Van Dooren, W., Vamvakoussi, X., & Verschaffel, L. (2013). Mind the gap-Task design principles to achieve conceptual change in rational number understanding. *Task design in mathematics education. Proceedings of ICMI Study 22*, *1*, 521-529.

Wademan, M. (2005). *Utilizing Development research to guide people capability maturity model adoption considerations.* Unpublished doctoral dissertation, Syracuse

Wassermann, J., Davis, C., & Astrab, D. P. (2007). *Overview of learning activities. 1-8. Activity design handbook. Faculty Guidebook*. Lisle: Pacific Crest. University. New York, USA.

Watson, A. (2008). Task transformation is the teacher's responsibility. In *Proceedings of the 32nd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 147-153).

Watson, A., & Mason, J. (2007). Taken-as-shared: A review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education*, *10*(4), 205-215.

Watson, A., & Ohtani, M. (2015). Task design in mathematics education: *An ICMI study.* Berlin: Springer.

Watson, A., Ohtani, M., Ainley, J., Frant, J. B., Doorman, M., Kieran, C., & Yang, Y. (2013). Introduction. In C. Margolinas (Ed.), *Task design in mathematics education. Proceedings of ICMI Study 22* (pp. 9–16). Oxford, England: ICMI.

Wiggins, G. (2012). Seven keys to effective feedback. *Feedback*, *70*(1), 10-16.

Yeşildere İmre, S. (2020). Design principles of mathematical activities. in Dede, Y., Doğan, M.F., Aslan-Tutak, F. (Ed), Activities and Applications in Mathematics Education. Ankara: Pegem Academy.

APPENDICES

APPENDIX 1: Indicators for Evaluation of Activity Script

Components	Definitions	Indicators
Objectives	Refer to the mathematical understanding or growth anticipated from the student(s) upon completion of the activity	**Clarity**: The activity's objectives are presented without any ambiguity or vagueness. **Comprehensibility**: The ease with which participants grasp and interpret statements about the activity's targeted output.
Materials	Encompass any kind of digital and tangible tools/objects aiding in instruction.	**Functionality**: Gauges the material's crucial role in achieving objectives and the impact of its absence. **Practicality**: Assesses the material's user-friendliness and efficiency.
Instructions	Delineate the task, convey expectations, and guide students towards the intended learning outcome.	**Easy-to-follow**: Instructions articulated in a clear and simple manner, devoid of extraneous or redundant information. **Relevance**: Instructions that directly relate to and serve the primary goal of the activity. **Alignment with the objective**: Guidelines that, when followed, steer learners toward achieving the target learning outcome.
Responsibility	Describes the duties and working styles that students are expected to adopt throughout the course of the activity.	**Role Definition**: Refers to the clear and understandable outlining of duties and expectations for students. **Student Engagement**: Emphasizes the importance of structuring roles to promote active participation.
Inclusion	Ensures that an activity caters to a diverse range of students at various developmental stages.	**Diverse Accessibility**: The attribute of an activity that ensures inclusivity, accommodating students from varied backgrounds such as socioeconomic, cultural, academic, gender, and ethnic origins.

APPENDIX 2: Indicators for Evaluation of Activity Implementation

Components	Definitions	Indicators
Attracting Student Attention	Entails guiding their engagement, sparking genuine interest, and highlighting the activity's relevance to enhance motivation and commitment."	**Interest Stimulation**: Strategies used by teachers to initiate and maintain student curiosity throughout the activity. **Significance Highlight**: The teacher's emphasis on the importance and relevance of the activity. **Felt Necessity**: Strategies aimed at making students feel the need to engage in the activity.
Organisation of Physical Conditions	Refers to the strategic arrangement of classroom furniture and resources to support optimal student engagement and activity implementation.	**Activity-Focused Arrangement**: The classroom setup, including furniture and seating, is tailored to the specific activity's needs. **Supportive Layout**: The physical layout supports the specific working styles prescribed by the activity.
Working in Accordance with Instructions	Refers to students' consistent alignment with and adherence to provided guidelines during an activity.	**Clear Communication of Instructions**: Instructions are delivered with clarity and precision to ensure students grasp them fully. **Student Adherence to Instructions**: Students actively engage in the activity as per the given guidelines.
Effective Use of Materials	Ensures that materials used in activities are effective tools for achieving mathematical outcomes, simultaneously facilitating task execution and enhancing understanding.	**Material alignment with objectives**: Materials directly serve the mathematical outcome without distracting. **Efficiency of Materials in Task Execution**: Materials provide a clear, direct route without causing chaos or confusion. **Smooth Material Integration**: Materials are introduced seamlessly, without interrupting the implementation or the flow.

Segmentation and Time Management	Refers to the structured organization and optimal use of time during various stages of an activity.	**Adequate Time Allocation**: This assesses whether time assigned for each segment of the implementation is both sufficient and effectively optimized to facilitate understanding and productivity. **Minimization of Time Loss**: This assesses whether time is wasted on irrelevant tasks or inefficient transitions that do not align with the intended learning outcome.
Classroom Management	Involves the teacher's active monitoring of student behaviours to ensure alignment with tasks and promptly intervening with appropriate strategies when disruptions or off-task behaviours arise.	**Monitoring**: Observing and ensuring that student actions and efforts are aligned with the task's objectives and are free from behavioural disruptions. **Intervention**: Implementing appropriate strategies to guide disruptive or struggling students back to the task.
Student Engagement	Encapsulates the depth and breadth of students' active involvement in activities.	**Active Participation**: Signifies students' genuine intellectual effort towards achieving the targeted learning outcomes. **Exchange of Ideas**: Refers to cultivating an environment that fosters sharing and collaboration among students. **Student Input**: Emphasizes integrating students' contributions (discussions, solutions, comments, explanations) into the progression of the activity.
Drawing Conclusions	Emphasizes the clear linkage between students' works and the intended mathematical outcomes, while ensuring these outcomes are understood and reflected upon by the students.	**Connection**: Focuses on the alignment of works that students have performed during the activity with the intended learning outcome. **Elucidation**: Pertains to the clarity and depth with which the learning outcomes are highlighted and discussed at the conclusion of the activity.

APPENDIX 3: Indicators for Evaluation of Mathematical Potential of Activity Script and Implemantation

Components	Definitions	Indicators
Depth	**S**ignifies the extent of understanding associated with concepts, principles, and generalizations underlying mathematical rules	**Elaboration**: Refers to the detailed understanding and thorough explanation of mathematical concepts and rules, emphasizing intricacies. –**Justification**: Covers students' capacity to ground their mathematical reasoning in core principles and laws, providing logical explanations for their ideas. –**Generalisation**: Involves the ability to extrapolate rules and trends from specific instances and apply them to various contexts.
Complexity	Refers to the understanding of relationships formed across time, between different perspectives, and among various disciplines	–**Mathematical connections:** Refers to establishing links among mathematical concepts, representations and solutions. –**Cross-disciplinary integration:** Relates mathematical ideas and applications to other academic disciplines, study fields or real-life situations.
Mathematical Focus	Refers to the clear and central emphasis of an activity on specific mathematical concepts or skills.	–**Mathematical Prominence:** Refers to the centrality and distinguishability of the mathematical objectives within an activity. –**Mathematical Thought Provocation:** A assesses the extent to which an activity encourages idea generation specifically aligned with its mathematical objectives.

APPENDIX 4: Criteria for Evaluation of the Activity Script Components

Components	Criteria			
	Very Low (0)	**Low (1)**	**Moderate (2)**	**High (3)**
Objectives	The activity's objective is presented with ambiguity, making it difficult for participants to grasp.	The activity's objectives are multiple and uncorrelated, making it challenging for participants to discern the primary intended outcome.	Even if the objective seems unclear at the start, following the instructions or completing the steps clarifies it.	The objective is clearly presented without ambiguity and easy to grasp.
Materials	The material is neither functional nor practical.	The material is practical but not functional.	The material is functional but not practical.	The material is both functional and practical.
Instructions	The instructions are not easy-to-follow, making it impossible to determine whether the activity's objective has been achieved or not.	The instructions are easy-to-follow, but there are issues or ambiguities that hinder the achievement of the activity's objective.	The instructions are easy-to-follow and offer the potential to achieve the activity's objective, but they contain some superfluous or unrelated steps.	The instructions are easy-to-follow, relevant, and fully aligned with achieving the objective.
Responsibility	Responsibilities assigned to students are not delineated or specified.	Student responsibilities are outlined, but they are presented in an ambiguous or unclear manner, leading to potential misunderstandings.	Student responsibilities are well-defined and clear, but the structure or presentation limits the active engagement and involvement of the students in the process.	Student responsibilities are clearly and precisely defined, encouraging active participation and engagement throughout the process.

Inclusion	The activity remains unable to accommodate the majority of the students.	The activity accommodates students with specific characteristics only.	While the activity accommodates most students, some from varied backgrounds remain uninvolved.	The activity ensures full diverse accessibility, accommodating all students.
Mathematical potential of the activity script				
Depth	Students solely identify or label information without deeper analysis or exploration.	Students recall basic mathematical knowledge and execute standard procedures without engaging in the underlying principles or justifications.	Students employ or apply mathematical knowledge in a manner that goes beyond simple recall but without requiring full justification or generalization.	Students deeply engage with mathematical knowledge, aiming for a through elaboration, justification, or generalization.
Complexity	The activity is structured without requiring any mathematical connections.	The activity presents opportunities for connections among mathematical concepts, solutions, or representations; however, these are either used in isolation without forging links if any, remain superficial.	The activity necessitates making connections among mathematical concepts, solutions, or representations, but interdisciplinary elements, if any, serve merely as context or setting.	The activity not only mandates mathematical connections but also enriches these with meaningful associations to other disciplines beyond mere context or setting.
Mathematical Focus	Mathematical objectives are unclear or indistinguishable, and tasks do not provoke thought or reflection on them.	Mathematical objectives are distinguishable, but tasks do not actively provoke thought or reflection on them.	Mathematical objectives are distinguishable; tasks provoke thought and reflection but give overt hints towards the desired outcome.	Mathematical objectives are distinguishable, and tasks effectively provoke critical thought and idea generation aligned with them.

APPENDIX 5: Criteria for Evaluation of Activity Implementation Components

Components	Criteria			
	Very Low (0)	Low (1)	Moderate (2)	High (3)
Attracting student attention	No visible attempts to stimulate student interest, emphasize the activity's relevance, or establish its necessity.	Relies mainly on pro-forma (standard/formal) expressions for engagement. Limited emphasis on the activity's importance and necessity.	Efforts go beyond mere pro-forma expressions to engage students, but the impact across the class remains limited.	Effectively stimulates genuine interest, emphasizes importance, and ensures students perceive the activity's necessity.
The organisation of physical conditions	The classroom setup noticeably hinders the activity's objectives, making the completion of the activity impossible.	While the activity proceeds, the environment's physical conditions cause significant challenges or interruptions during the execution.	The classroom's physical conditions may not be fully tailored for the specific activity, but they do not obstruct its execution.	The classroom's physical setup perfectly aligns with the activity's requirements, ensuring a smooth execution.
Working in accordance with instructions	The instructions were either not communicated or were misunderstood, leading to unrelated tasks being performed or the activity being incomplete.	Instructions were provided, yet a significant portion of the class struggled to comprehend and/or follow them, indicating gaps in the communication or understanding	The teacher clearly communicated the instructions. However, a few students exhibited challenges in aligning their actions with the given guidelines	The teacher explicitly conveyed the instructions, and all students engaged in the activity according to the provided guidelines.
Effective Use of Materials	The materials do not align with the mathematical objective and cause significant disruptions in the implementation.	While the materials align with the mathematical objective, they cause confusion and problems severe enough to hinder the implementation.	The materials align with the mathematical objective and cause only minor issues that do not significantly disrupt the implementation.	Materials perfectly align with the mathematical objective, and no material-related problems are observed during the implementation.

Segmentation and time management	Time spent on the activity is predominantly occupied with unrelated tasks, resulting in the activity being uncompleted.	There are considerable instances where time is wasted on works that don't support the progress in the mathematical outcome.	Some segments of the activity are either over-extended or too rushed, which may hinder the students' grasp of the mathematical outcome.	The time set for each stage and the entire activity effectively supports students' progress in the mathematical outcome, avoiding time-wasting tasks or delays.
Classroom management	The teacher often overlooks disruptive behaviours with little or no intervention.	The teacher recognizes most disruptions but interventions often do not fully resolve them.	The teacher actively identifies disruptions and while interventions are mostly successful, some students still face brief task interruptions.	Disruptions, if present, are promptly addressed, ensuring a seamless continuation of the task for all students.
Student Engagement	Majority of students remain passive observers with minimal exchange of ideas or integration of their inputs.	Activity is driven by a select few students' participations or contributions, with little influence on achieving the targeted outcome.	While most students participate and share ideas, their collective inputs had a limited impact on the progression of targeted outcome.	Students extensively participate and share ideas, with their inputs significantly shaping the progression of the targeted outcomes.
Drawing conclusions	Activity concludes without drawing any conclusion, lacking connection or elucidation of the intended mathematical outcome.	Activity is concluded, but does not sufficiently emphasize the mathematical outcome or relate it to the works done.	While the works are related to the mathematical outcome, there is no verification of its clarity or how well students understood it.	The works are clearly connected with the mathematical outcome, and feedback confirms it has become clear for students.

Mathematical Potential of Implementation				
Depth	Students solely identify or label information without deeper analysis or exploration.	Students recall basic mathematical knowledge and execute standard procedures without engaging in the underlying principles or justifications.	Students employ or apply mathematical knowledge in a manner that goes beyond simple recall but without requiring full justification or generalization.	Students deeply engage with mathematical knowledge, aiming for a through elaboration, justification, or generalization.
Complexity	The activity is structured without requiring any mathematical connections.	The activity presents opportunities for connections among mathematical concepts, solutions, or representations; however, these are either used in isolation without forging links or the connections, if any, remain superficial.	The activity necessitates making connections among mathematical concepts, solutions, or representations, but interdisciplinary elements, if any, serve merely as context or setting.	The activity not only mandates mathematical connections but also enriches these with meaningful associations to other disciplines beyond mere context or setting.
Mathematical Focus	Mathematical objectives are unclear or indistinguishable, and tasks do not provoke thought or reflection on them.	Mathematical objectives are distinguishable, but tasks do not actively provoke thought or reflection on them.	Mathematical objectives are distinguishable; tasks provoke thought and reflection but give overt hints towards the desired outcome.	Mathematical objectives are distinguishable, and tasks effectively provoke critical thought and idea generation aligned with them.